# MERGERS

Farasat A.S. Bokhari
University of East Anglia
f.bokhari@uea.ac.uk

Core Industrial Organisation Course for
Postgraduate Certificate in Competition and Regulatory Policy

Fall 2021

We will discuss unilateral effects of mergers, starting with some basic theory and then discuss specific empirical techniques and focus on the predicted effects on prices, outputs and welfare*

- Horizontal Mergers
  - Mergers in homogeneous markets (w/ Cournot competition)
    - Why HHI? – Link to market power
    - Merger 2 to 1 (duopoly to monopoly)
    - Merger 3 to 2 (oligopoly)
  - Mergers in differentiated products (w/ Bertrand competition)
    - Screening Tools
    - Diversion Ratios and UPP
    - Merger simulation
- Vertical Mergers
  - Double marginalization and other externalities
  - Foreclosure and other concerns
- Appendix – Merger simulation (numerical example w./ linear demand)

*These slides draw in part from Bruce Lyon's lecture in the MSc IO sequence.

# Horizontal Mergers

- Efficiency (in production, marketing, R&D)
  - Economies of scale
  - Economies of scope (product range)
  - Other synergies
  - Coordination & reduced transaction costs (mostly vertical)
- Corporate control
  - No family successor
  - Managerial interests
  - Market for corporate control
- Market power
  - Firms want to gain market power and increase prices (and profits)
  - Sometimes incidental to efficiency/control motives

- Standards in competition law
  - Substantial lessening of competition (SLC) in US, UK
  - Significant impediment to effective competition (SIEC) in EU
- What does ‘less competition’ mean?
  - Market structure – e.g. number of firms, market shares, concentration ratio, HHI; entry barriers
  - Behaviour of firms – e.g. fighting for customers by offering low price & high quality; product range available to customers; tacit collusion
  - Expected outcomes – e.g. prices, innovation
  - How will the changes in structure of the market effect the outcomes?
- Economics effects based interpretations
  - Consumer welfare standard (CWS): Consumer Surplus ($CS$) only
  - Total welfare standard (TWS): $CS + \pi$ (where $\pi$ refers to the profits of the merging parties and that of the non-merging parties)

- Consumer welfare standard
  - Merger must not harm consumer welfare
  - Price must not be expected to rise, at least without offsetting quality improvements
  - No detriment on quality, range, service, innovation
- Total welfare standard
  - Merger is allowed if rise in profits at least offsets fall in consumer surplus
  - E.g. Small price rise may be acceptable if merger creates sufficient efficiencies

- We often focus on the Herfindahl-Hirschman Index (HHI) as a rough indicator of market power, and how much it will change under a proposed merger

$$HHI = \sum_{i}^{N} (100s_i)^2$$

where $s_i$ is the share of the $i$th firm (for a monopoly it takes a value of 10,000 and for 10 firms equal size is 1,000 or in general $100^2/N$)

- Competition authorities use it as an initial screening device
  - Example - US DOJ considers HHI between 1,500-2,500 to be moderately concentrated and above 2,500 to be highly concentrated and mergers that increase HHI by more than 200 points in highly concentrated markets are presumed to *enhance market power*
  - Similar role of HHI in some other jurisdictions as well

- How does HHI link to measures of market power?
  - One measure is the price cost margin, or the Lerner index $L = \frac{p-c}{p}$, where p and c are the price and marginal costs

- Let there be $N$ firms all producing a homogenous product, and where firms compete in quantity (Cournot competition)
  - Let inverse market demand be $p(Q)$
  - Each firm produces $q_i$ at marginal cost $c_i$ and total demand $Q = \sum_i q_i$
  - Then profit is $\pi_i = p(Q)q_i - c_i q_i$
  - First order conditions (henceforth FOC) give

$$\frac{\partial \pi_i}{\partial q_i} = \frac{dp}{dQ} q_i + p(Q) - c_i = 0$$

which, after a little algebra, gives the relationship

$$\frac{p - c_i}{p} = \frac{s_i}{\eta}$$

where $s_i = q_i/Q$ is the share, and $\eta = -d \ln Q / d \ln P$ is the elasticity of demand

  - Thus, firms' profit margin is equal to their market share divided by the elasticity of demand

- Now if we multiply each side by $s_i$ and add up over all firms then

$$\frac{p - c_i}{p} = \frac{s_i}{\eta}$$

gives

$$s_1 \frac{p - c_1}{p} + \ldots + s_n \frac{p - c_n}{p} = \sum_i^n \frac{s_i s_i}{\eta} = \frac{\text{HHI}}{\eta}$$

where $\text{HHI} = \sum_i^n s_i^2$ (where we have dropped multiplication of $s_i$ by 100)

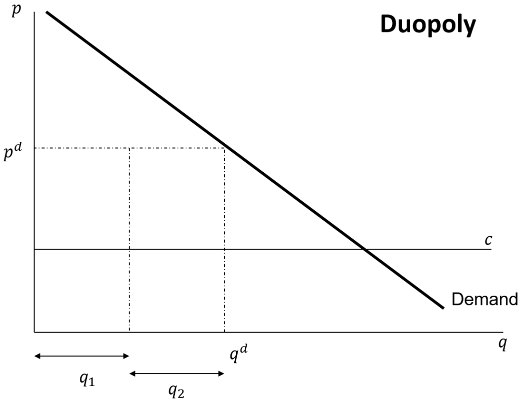
- Under such a model, HHI is directly related to weighted average of price cost margins of each firm (a measure of market power)
  - When all firms have equal share, $\text{HHI} = \sum_i^n s_i^2 = N(1/N)^2 = 1/N$
  - If shares are unequal, $1/\text{HHI}$ can be thought of as effective number of firms in the industry

- When two (or more) firms merge, we add up their shares as one new firm and compute HHI before and after the merger ($\Delta = HHI^0 - HHI^1$)
  - This can give us an initial rough idea about how the merger may effect competition
  - However this is not based on an equilibrium analysis – firms react and adjust output
  - In Cournot competition, after the merger we expect the merging firms to *reduce* output, and the competitors to *increase* their output
  - A simple addition of shares by merging parties does not capture this change in shares

- We highlight these facts by analyzing a $2 \rightarrowtail 1$ and a $3 \rightarrowtail 2$ merger under Cournot competition with homogenous products
  - An important element in mergers in to also consider how the competitors will react to the merging parties actions – if we ignore the latter, it is a partial equilibrium analysis
  - Thus a $2 \rightarrowtail 1$ merger can also be thought of as if we were ignoring actions by other non-merging firms

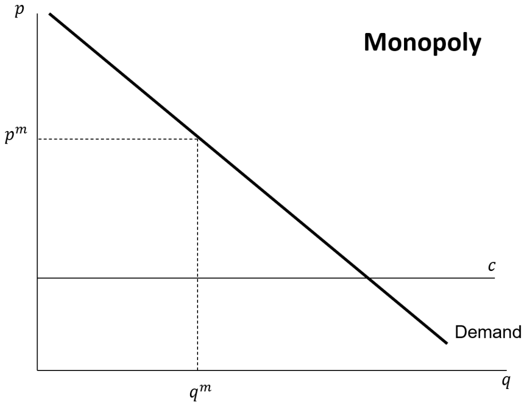
- To predict the effect of a merger, we need to compute the pre and post *best response functions* of each firm
- These could be in quantity setting Cournot industries
(or in price setting Bertrand competition which we will focus on later)
- We begin with a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- We will find that (there's a bad moon on the rise, all right!)
  - absent any efficiency gains, price increases, output and consumer welfare decrease
  - the loss in consumer surplus is greater than an increase in profits
  - thus, both the consumer surplus and total surplus (= cs + profits) decrease
  - With efficiency gain, total welfare may increase ('efficiency defence')

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Say there are no merger specific efficiencies



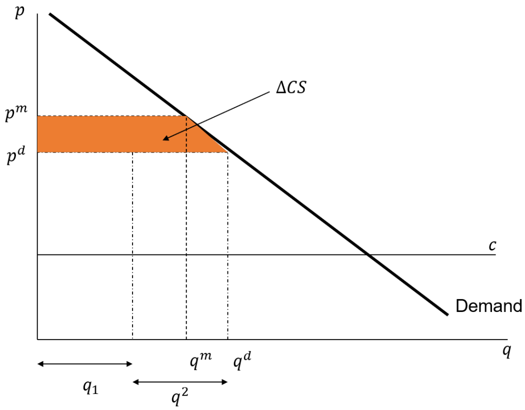

- Let the duopoly price be $p^d$ where each firm sells $q_1 + q_2$ equal to market supply of $q^d$

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Say there are no merger specific efficiencies



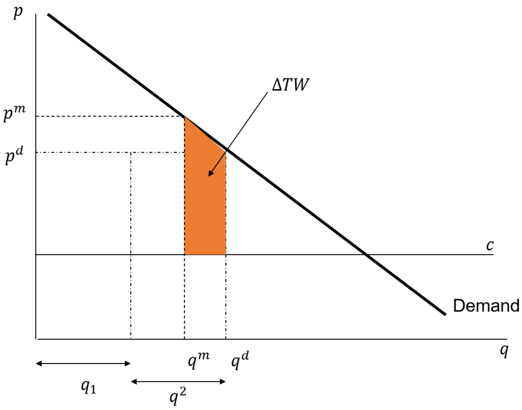

- Let the duopoly price be $p^d$ where each firm sells $q_1 + q_2$ equal to market supply of $q^d$
- After merging, they set monopoly output and price (i.e., where MR=MC)

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Say there are no merger specific efficiencies



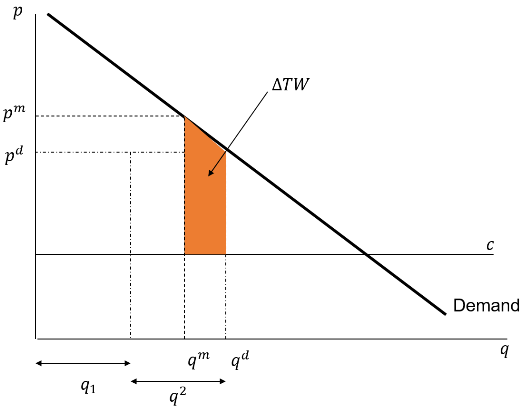

- Let the duopoly price be $p^d$ where each firm sells $q_1 + q_2$ equal to market supply of $q^d$
- After merging, they set monopoly output and price (i.e., where MR=MC)
- CS reduces

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Say there are no merger specific efficiencies



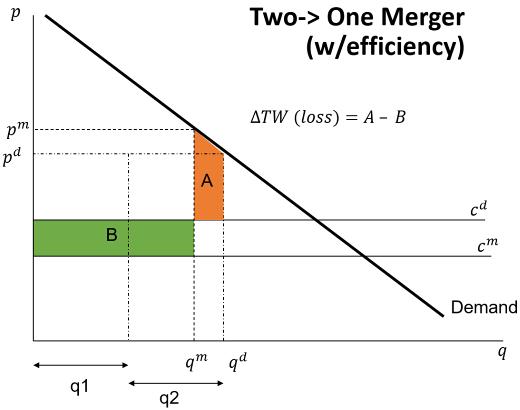

- Let the duopoly price be $p^d$ where each firm sells $q_1 + q_2$ equal to market supply of $q^d$
- After merging, they set monopoly output and price (i.e., where MR=MC)
- CS reduces as does TW

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Say there are no merger specific efficiencies




- Combined firm reduces output
- Higher price
- Consumers lose more surplus than firms gain in profit
- *Consumer welfare standard* and *total welfare standard* both would lead to the same conclusions

- Consider a simple model of homogeneous products where two firms *compete in quantity* and then merge to a monopoly
- Now allow for an efficiency gain – marginal cost decreases




- Combined firm reduces output (probably)
- Higher price (maybe)
- Profit rise (yes)
- Resource savings
- Total welfare loss = $A - B$; it could be negative, so total *could* increase

- Some main points of $2 \rightarrowtail 1$
  - If there is no efficiency gain, output decreases, price increases
  - Consumer welfare decreases and profits increase
  - But profits increase by less than the loss in consumer surplus
  - If there is an efficiency gain, total welfare could increase
- Next let's look at $3 \rightarrowtail 2$

Important new effects arise when merger is from oligopoly to oligopoly

- Private effects
  - Mergers without efficiencies may no longer be profitable (known as the *merger paradox*)
- Competitive effects
  - Unilateral effects (= independent action by merged firms) or
  - Coordinated effects (= enhanced incentive for tacit collusion with 'outside' rivals)
- Welfare effects
  - Profitable mergers are less likely to harm total welfare (If associated with substantial efficiencies )
- These changes are due to the importance of 'outsiders' (non-merging firms) in the market

Importance of non-merging firms: *unilateral effects*

- Rivals have an incentive to fill (part of) the void created by merging firms reducing output
- In a Cournot oligopoly, firms set output – so how will output of independent rivals change post-merger?
    - Cannot ignore the outsides actions (in $2 \rightarrowtail 1$ not an issue as no outsiders)
    - 'Market equilibrium' approach to merger analysis
- Merger paradox: Cournot mergers increase market power but are typically unprofitable for merging parties unless major efficiencies

- Note that in a homogenous product Cournot industry with linear (inverse) demand curve

$$p(Q) = a - bQ$$

*equilibrium* prices and quantity are given by

$$q_i = \frac{a-c}{b(N+1)} \qquad p = \frac{a+Nc}{N+1}$$

where $Q = \sum_i^N q_i$ is the aggregate demand, $c$ is the marginal cost, $N$ is the number of (symmetric) firms, and $a$ and $b$ are demand parameters†

  - As $N$ increases, prices fall ($dp/dN < 0$)
  - But the model also has some other curious predictions
  - Merging firms will reduce their combined output, the competitor increases its output, and unless there is an efficiency gain, the merging firms have lower combined profits ('merger paradox')!

---

†The equilibrium values are computed using *best response functions* of each firm, which in turn are derived using FOCs

- Example – consider a three to two merger and let $a = 1$, $b = 1$ and $c = 0$
- Then, before merger

$$q_i^0 = \frac{a-c}{b(N+1)} = \frac{1}{4} \qquad p^0 = \frac{a+Nc}{N+1} = \frac{1}{4}$$
$$\pi_i^0 = p^0 q_i^0 - c q_i^0 = \frac{1}{16}$$

(superscript 0 is to indicate before merger equilibrium values) and the combined outputs and profit of merging firms 1 and 2 are

$$\pi_1^0 + \pi_2^0 = \frac{1}{8} \qquad q_1^0 + q_2^0 = \frac{1}{2}$$

while the industry output is $Q^0 = \frac{3}{4}$

- Note also that the HHI before merger is

$$HHI^0 = \sum_i^N (100 * s_i)^2 = (100 * \frac{1}{3})^2 + (100 * \frac{1}{3})^2 + (100 * \frac{1}{3})^2$$
$$= 3,333$$

(highly concentrated as above 2,500 per DOJ guidelines)

- The non-simulated predicted changes in HHI (based on adding up of shares of merging firms) is

$$HHI^{1'} = \sum_{i}^{N}(100 * s_i)^2 = (100 * \frac{1}{3})^2 + (100 * \frac{2}{3})^2$$
$$= 5,555$$

(the superscript $1'$ indicates post merger predicted value without simulation)

- The 'delta' is 2,222 and the merger would be scrutinized due to high initial values and high delta (delta more than 200)

- Compare this to the new equilibrium values after the merger

- Post merger equilibrium values are

$$q_i^1 = \frac{1}{3} \qquad p^1 = \frac{1}{3} \qquad \pi_i^1 = \frac{1}{9}$$

- The implied HHI (based on new equilibrium) is

$$HHI^1 = \sum_i^N (100 * s_i)^2 = (100 * \frac{1}{2})^2 + (100 * \frac{1}{2})^2 = 5,000$$

where $HHI^1 = 5,000 < 5,555 = HHI^{1'}$

- Other predicted changes are
  - The price *increases* from $p^0 = 1/4$ to $p^1 = 1/3$
  - Industry output *decreases* from $Q^0 = 3/4$ to $Q^1 = 2/3$
  - The merging firms' *combined* output *decreases* from $q_1^0 + q_2^0 = 1/2$ to $q_{12}^1 = 1/3$
  - The merging firms' *combined* profit *decreases* from $\pi_1^0 + \pi_2^0 = 1/8$ to $\pi_{12}^1 = 1/9$
  - The non-merging firm's output and profit *increase* (output increases from $1/4$ to $1/3$ and profit increases from $1/16$ to $1/9$)

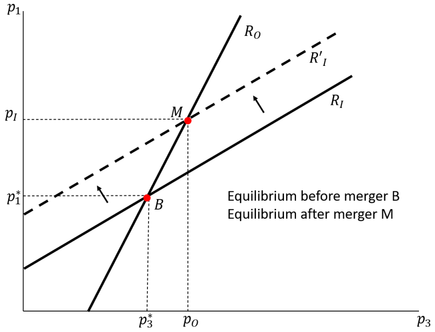
- While part of the peculiarity of results is due to the simple linear nature of the demand curves and assumed symmetries in cost between merging firms, the underlying issue is of assumed form of competition
  - Absent any efficiencies, Cournot model always leads to lower profits for the merging firms, unless two-to-one monopoly creating merger (in price competition models, all mergers will be potentially profitable)
  - Merging parties restrict output, but since firms choose quantity in Cournot competition, these are games of strategic substitutes (downward sloping best response functions), and so competing firms react by increasing their output
  - Either the model is not appropriate, or perhaps consistent with only an efficiency motivation for a merger
  - Farrell and Shapiro (1990) show prices in Cournot models always increase unless there are efficiency gains – cost of merged firms must be less than the smaller of the merging firms costs, i.e., $c_{12}^1 < \min\{c_1^0, c_2^0\}$) they also discuss asymmetries in costs and economies of scale and welfare effects

- Why merger paradox may not apply
  - Efficiencies
    - Elimination of fixed cost duplication – no price effect but good for profits
    - Merging firms have 'twice as much' capital (e.g. capacity) – capital reduces marginal cost $\Rightarrow$ merger more likely to be profitable
    - Synergies $c_{12}^1 < \min\{c_1^0, c_2^0\}$
  - Price competition
    - Bertrand price competition with product differentiation
    - Prices are strategic complements (upward sloping best response functions) – rivals raise price in response to merger
    - Merge much more likely to be profitable
  - Coordinated effects
    - We have been considering unilateral effects – merger may lead to conditions so that tacit collusion more likely

## Mergers in differentiated products (w/ Bertrand competition)

- Most mergers involve differentiated products
  - Motta section 5.4 provides an alternative model
  - Three firms, linear demand, differentiated products
  - 3 $\rightarrowtail$ 2 merger and no efficiencies
  - Firms compete in prices
  - Prices are strategic complements (upward slopping best response functions)
  - New equilibrium such that

- Best response function of the merging parties (‘inside firms’ relative to non-merging outside firm) shifts up
- Consumer surplus decreases
- Total welfare (profit + cs) decreases



- Most mergers involve unilateral effects and differentiated products
  - Combining shares when products are not close substitutes need not change prices
  - Combining small market shares in the same niche can raise prices
  - Shares can overstate degree of competition if products are not close substitutes, or understate if they are close substitutes
- Dissatisfaction with traditional market shares based approach
  - Particularly misleading under product differentiation
- Farrell and Shapiro advocate UPP as an alternative (Farrell and Shapiro, 2010)
  - Straight forward and requires less data
  - Aims to answer: Does merger create incentive to raise price?
  - Requires measures of margins and diversion ratios
  - But does not attempt to predict by how much

- Screening tools for unilateral effects
  - **Diversion ratios** – Measure closeness of competition among merging firms
  - **Upward Pricing Pressure (UPP)** – Measure the effect of internalizing this diversion

  - **Merger Simulations** – Predicting future based on estimates of structural parameters (I have this bridge in Brooklyn ...)

  - Critical Loss Analysis – What sales loss would negate the benefit of price increase
  - Indicative Price Rise – Predicting price increase from diversion ratios and margins
  - Reduced Form Price Regressions – Regress price as a function of number of competitors and other controls

We will focus on just the first three of these

- Diversion Ratio from A to B
- Measures the fraction of consumers currently consuming A that switch to B in response to a price increase in A
- Intuitively fraction of A's consumers that have B as their second choice
- Technically

$$D_{AB} = -\frac{\frac{\partial q_B}{\partial p_A}}{\frac{\partial q_A}{\partial p_A}} = -\frac{q_B}{q_A}\frac{\eta_{AB}}{\eta_{AA}}$$

  where $\eta_{AA}$ and $\eta_{AB}$ are the own and cross-price elasticities $\eta_{kj} \equiv \frac{\partial q_j(\cdot)}{\partial p_k}\frac{p_k}{q_j(\cdot)}$

- Uses sales data to estimate elasticities
- Can use survey data to get estimates of second choice
  - What would you do if price of A went up by 5-10%?
  - What would you do if A is not available?

- It can be shown that

$$\Delta \frac{p_A - c_A}{p_A} = \Big( - \frac{1}{\eta_{AA}} \Big) \Big( \frac{D_{AB}}{1 - D_{AB}} \Big)$$

- Market power increases with merger if
  - Own demand elasticity is low
  - Diversion ratios are high

Why use Diversion Ratios and UPP?

- Suppose firms making products A and B merge
  - Effects of competition between brands is **internalised**
  - A's pricing now takes account of B's profit (internalization)
  - If $p_A$ is increased, some consumers switch to product B
  - Proportion switching from $A \rightarrowtail B = D_{AB}$ – 'Diversion ratio from A to B'
  - B earns margin $(p_B - c_B)$ on each of these switches ($c_B$ is the marginal cost of B)
  - Intuition: 'externality' of A's price rise benefiting rivals is partially internalised by merger – this creates incentive to raise price

- If products are substitutes, must always expect prices to rise in absence of marginal cost savings
  - UPP formula offers merging firms the benefit of some small assumed efficiencies $E_A (< 0)$ on A's marginal cost $(= c_A)$

What is Internalization?

- Suppose firms making products A and B merge
- Before merger, if A increases price
  - +ve: increase in profit due to increase in markup $(p_A - c_A)$
  - -ve: loss in profit as some leave the market
  - -ve: loss in profit as some purchase product B instead
- After merger, if A increases price
  - +ve: increase in profit due to increase in markup $(p_a - c_A)$
  - -ve: loss in profit as some leave the market
  - +ve: **increase in profit as some purchase product B instead** $(p_B - c_B)$
- Capture by B is the internalization and depends on diversion ratio from A to B

What is Internalization?

- Before merger, firm A cares about maximizing own profits only

$$\max_{p_A} \pi_A = (p_A - c_A) q_A(p_A, p_B)$$

FOC:

$$\frac{\partial \pi_A}{\partial p_A} = q_A(p_A, p_B) + (p_A - c_A)\frac{\partial q_A}{\partial p_A} = 0$$

- After merger, firm A maximizes joint profits

$$\max_{p_A} \pi_A + \pi_B = (p_A - c_A) q_A(p_A, p_B) + (p_B - c_B) q_B(p_A, p_B)$$

FOC:

$$\frac{\partial \pi_A + \pi_B}{\partial p_A} = q_A(p_A, p_B) + (p_A - c_A)\frac{\partial q_A}{\partial p_A} + (p_B - c_B)\frac{\partial q_B}{\partial p_A} = 0$$

What is Internalization?

- Note that at premerger equilibrium price

$$\frac{\partial \pi_A + \pi_B}{\partial p_A} = q_A(p_A, p_B) + (p_A - c_A)\frac{\partial q_A}{\partial p_A} + (p_B - c_B)\frac{\partial q_B}{\partial p_A}$$
$$= 0 + (p_B - c_B)\frac{\partial q_B}{\partial p_A} > 0$$

- Thus, there is incentive to increase price – strength of effect depends on
  - Market power before merger : $(p_B - c_B)$
  - How close are A and B as substitutes : $\frac{\partial q_B}{\partial p_A}$

The UPP formula

- Offers merging firms the benefit of some small assumed efficiencies $E_A (< 0)$ on A's marginal cost ($= c_A$)

$$UPP = D_{AB}(p_B - c_B) - \Delta c_A$$

- There is upward pricing pressure (and the merger may harm competition) if $UPP > 0$
- UPP is margin on acquired product multiplied by diversion ratio from A to B minus the efficiency gain
- Note – the formula assumes price setting but not full equilibrium (i.e. other prices including $p_B$ are held constant)

A simple example

- The table below shows sales at current prices $q(p^0)$, and sales if firm A raised its prices by 5% i.e., $q(p^1)$
- Which merger is most profitable/harmful to competition?

| Firm | $q(p^0)$ | $q(p^1)$ |
|---|---|---|
| A | 100 | 80 |
| B | 100 | 115 |
| C | 100 | 103 |
| D | 100 | 102 |

- Computing elasticities and diversion ratios

| Firm | Elasticity $\eta_{Aj}$ $\left(\frac{\Delta q_j/q_j}{\Delta p_A/p_A}\right)$ | Diversion $D_{Aj}$ $-\frac{q_j \eta_{Aj}}{q_A \eta_{AA}}$ |
|---|---|---|
| A | $\frac{-.2}{.05} = -4$ | N/A |
| B | $\frac{.15}{.05} = 3$ | $-\frac{3}{(-4)} = 0.75$ |
| C | $\frac{.03}{.05} = 0.6$ | $-\frac{0.6}{(-4)} = 0.15$ |
| D | $\frac{.02}{.05} = 0.4$ | $-\frac{0.4}{(-4)} = 0.10$ |

- UPP for alternative mergers of A with firm $j$

$$UPP = D_{Aj}(p_j - c_j) - \Delta c_A$$

- Assume $(p_j - c_j) = .2$ and $\Delta c_A = 5\%$
  - with $j = B$: $UPP = .75(.20) - .05 = .10 = 10\% > 0$
  - with $j = C$: $UPP = .15(.20) - .05 = -.02 = -2\% < 0$
  - with $j = D$: $UPP = .10(.20) - .05 = -.03 = -3\% < 0$

General Idea

- Uses a model of demand and competition to predict the effect of merger
- Typically used to study unilateral effects
  (can also be used to assess potential for coordinated affects)
- Need information on
  - Demand for all products
  - Marginal costs
  - Nature of competition
- Can be divided into two main parts
  - Front end – estimate demand and cost parameters
  - Back end – use these as input in a model of competition to predict effect of merger (or other changes)

- Commonly used demand systems
    - Linear/Log-linear
    - Almost Ideal Demand System (AIDS)
    - Logit/Nested Logit
    - Random Coefficients Logit
- Pros and cons – models differ in flexibility for own- and cross-price elasticities, requirements on data, and difficulty of estimation
    - Linear and AIDS – flexible and can give negative cross-elasticities (complements), but difficult to estimate if too many products (the 'dimensionality curse')
    - Logit – easy to estimate, suffers from the 'independence of irrelevant alternatives' (IIA) problem, and if shares are small, own elasticity is proportional to price
    - Random coefficients and its variants difficult to estimate under strict time restrictions
- Endogeneity – models must account for simultaneity of price and quantity

Models of Competition

- What are the strategic variables?
  - Prices, quantities, quality, advertising
- How do firms set their values?
  - Cooperatively or non-cooperatively
  - Simultaneously or sequentially
- What is the equilibrium concept?
  - Typically Nash equilibrium
- We will focus on differentiated products Bertrand competition where
  - Firms move simultaneously to set prices
  - Outcome is via Bertrand-Nash equilibrium

- Costs can be obtained from independent sources (e.g. firms accounts, industry reports)
- Can also be backed out from demand model when combined with a model of competition such as Bertrand-Nash equilibrium
- Intuition from a monopolist's problem ...

- Consider a monopolist's profit maximization problem,

$$\max_p pq(p) - TC(q(p))$$

FOC imply

$$q(p) + p\frac{\partial q(p)}{\partial p} - c(q(p))\frac{\partial q(p)}{\partial p} = 0$$

where $\partial TC(q(p))/\partial q = c(q(p))$ is the marginal cost

- At the optimal price

$$(p^* - c(q(p^*))) = -\frac{q(p)}{\partial q(p)/\partial p}\bigg|_{p=p^*}$$

or equivalently,

$$\frac{p^* - c(q(p^*))}{p^*} = -\frac{1}{\eta(p^*)}$$

where $\eta(p^*)$ is the price elasticity of demand

- Inferring costs:

$$\frac{p^* - c(q(p^*))}{p^*} = -\frac{1}{\eta(p^*)}$$

- If the monopolist is pricing optimally, then an estimate via demand estimation (or prior knowledge) of elasticity of demand $\eta$ and observed price allows us to infer marginal cost $c$ from the formula above
- The equation can be rewritten as **price is equal to marginal cost plus a markup**

$$p = c + \frac{1}{(\partial q(p)/\partial p)} q(p)$$

  - The markup depends on the curvature of the demand curve (if demand is perfectly elastic, as in the case of the perfect competition, then p = c)
- Idea extends to oligopoly as we will see (and as we shall see, this equation in multiprouct/multifirm context is used to predict prices under merger)

- Let there be $J$ differentiated products and $F$ firms and where the $f$-th firm produces a subset $\mathfrak{F}_f$ of the $J$ products
- Let the demand for the $j$-th product be given by

$$q_j = q_j(\mathbf{p})$$

  where $\mathbf{p}$ is a vector of all related prices (could be any of the demand functions we discussed earlier)
- The the $f$-th firm maximizes its joint profit over products that it produces

$$\Pi_f = \sum_{k \in \mathfrak{F}_f} (p_k - c_k) q_k(\mathbf{p})$$

  where $c_k$ is the marginal cost of the $k$-th product, typically assumed constant over the relevant range, and the sum is over all the products owned by firm $f$

- For firm $f$, the first order conditions for profit maximization (Nash-Bertand competition) are

$$q_j(\mathbf{p}) + \sum_{k \in \mathfrak{F}_f} (p_k - c_k) \frac{\partial q_k(\mathbf{p})}{\partial p_j} = 0 \qquad \text{for all } j \in \mathfrak{F}_f$$

- Let $\boldsymbol{\Theta}$ be a 1/0 joint "ownership" so that terms $\theta_{jk}$ (row $j$ column $k$) equal 1 if products $j$ and $k$ belong to the same firm and 0 otherwise (and 1 on the leading diagonal)
- Then we can re-write the FOC equations above for each firm $f$ as

$$q_j(\mathbf{p}) + \sum_{k=1}^{J} \theta_{kj} (p_k - c_k) \frac{\partial q_k(\mathbf{p})}{\partial p_j} = 0 \qquad \text{for all } j \in \mathfrak{F}_f$$

which will give us a total of $J$ such equations

- Example: firm 1 owns products 1,2, firm 2 owns products 3,4 and firms 3 and 4 own products 5 and 6 respectively

$$q_1 + \theta_{11}(p_1 - c_1)\frac{\partial q_1}{\partial p_1} + \ldots + \theta_{61}(p_6 - c_6)\frac{\partial q_6}{\partial p_1} = 0$$
$$q_2 + \theta_{12}(p_1 - c_1)\frac{\partial q_1}{\partial p_2} + \ldots + \theta_{62}(p_6 - c_6)\frac{\partial q_6}{\partial p_2} = 0$$
$$\vdots$$
$$q_6 + \theta_{16}(p_1 - c_1)\frac{\partial q_1}{\partial p_6} + \ldots + \theta_{66}(p_6 - c_6)\frac{\partial q_6}{\partial p_6} = 0$$

where note that only those terms survive where $\theta_{kj} \neq 0$

Rewrite in matrix notation as

$$\mathbf{q} - \mathbf{\Omega}(\mathbf{p} - \mathbf{c}) = \mathbf{0} \qquad \text{where} \qquad \Omega_{jk} = -\theta_{kj}\frac{\partial q_k(\mathbf{p})}{\partial p_j}$$

- Equivalently, given a demand system $q_j = D_j(\mathbf{p})$, if the matrix of slope coefficients $\frac{\partial q_j(\mathbf{p})}{\partial p_k}$ (row $j$ column $k$) is given by $\mathbf{B}$, then

$$\boldsymbol{\Omega} = -\boldsymbol{\Theta} \cdot \mathbf{B}'$$

(note: the symbol $\cdot$ is *element by element multiplication* and not the usual matrix multiplication)

- The quantity equation above can be rewritten as the price markup equation

$$\mathbf{p} = \mathbf{c} + \boldsymbol{\Omega}^{-1}\mathbf{q}(\mathbf{p})$$

(compare this to the monopolist's equation on **slide 41** – same/similar)

- This price equation, along with a demand system equations $q_j = D_j(\mathbf{p})$ jointly determines equilibrium prices and quantities and are at the heart of merger simulation

- Given estimates of demand functions, information about ownership, and observed prices and quantities, we can back out markups and marginal costs

$$\mathbf{p} = \mathbf{c} + \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$$
$$\Rightarrow$$
$$\mathbf{c} = \mathbf{p} - \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$$

- For merger simulations we change the ownership matrix $\mathbf{\Theta}$ and re-solve for prices using the equations $\mathbf{p} = \mathbf{c} + \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$ and $q_j = D_j(\mathbf{p})$

- Step 0a: Estimate the demand system $q_j = D_j(\mathbf{p})$ and obtain $\mathbf{B}$ the matrix of slope coefficients (or use previous studies); i.e. $B_{jk} = \frac{\partial q_j(\mathbf{p})}{\partial p_k}$
- Step 0b: Construct $\boldsymbol{\Omega}_0 = -\boldsymbol{\Theta}_0 \cdot \mathbf{B}'$ using pre-merger ownership matrix $\boldsymbol{\Theta}_\mathbf{0}$
- Step 1: Given data on price and quantity back out estimates of marginal cost $\widehat{\mathbf{c}} = \mathbf{p}_0 - \boldsymbol{\Omega}_\mathbf{0}^{-1}\mathbf{q}_\mathbf{0}(\mathbf{p}_\mathbf{0})$ (unless available from outside)
- Step 2: Construct the new ownership matrix $\boldsymbol{\Theta}_\mathbf{1}$
  (optionally, adjust mc of merging parties as necessary)
- Step 3: Compute the new equilibrium price $\mathbf{p}_\mathbf{1}^*$ using the equation $\mathbf{p}_\mathbf{1}^* = \widehat{\mathbf{c}} + \boldsymbol{\Omega}_\mathbf{1}^{-1}\mathbf{q}(\mathbf{p}_\mathbf{1}^*)$
  - If the demand system is linear we get a closed form solution for price and quantity
  - If not linear, will need to search for new price equilibrium using numerical methods
  - Given type of demand model, can iteratively search for $\mathbf{p}_\mathbf{1}^*$ such that
    $|\mathbf{p}^{(h+1)} - \mathbf{p}^{(h)}| < \epsilon$ and where
    $\mathbf{p}^{(h+1)} = \widehat{\mathbf{c}} + \boldsymbol{\Omega}_\mathbf{1}^{-1}(\boldsymbol{p}^{(\boldsymbol{h})})\boldsymbol{q}(\boldsymbol{p}^{(\boldsymbol{h})})$
    and $h$ is the iteration loop

- Data requirements can be high
  - Sales data including product characteristics, cost data and/or data on inputs that affect cost (additional supply side estimation)
  - Expertise in demand estimation
- Sensibility and sensitivity checks
  - Do elasticities, margins, marginal costs seem reasonable? Do they match some known outside information?
  - How much do they change with demand specification?
  - Do the assumptions made for the model make sense?
- Proceed with caution
  - They can provide reasonable predictions but require great care
  - Predictions are sensitive to modelling assumptions
  - Perhaps use it as internal screen that complements other qualitative work

# Vertical Mergers

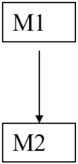
- Vertical control either by ownership (= vertical integration) or by contract (= vertical restraint)
  - Vertical restraints include exclusive dealing, exclusive territories selective distribution, resale price maintenance, quantity forcing, quantity rebates
  - Vertical integration or vertical restraint are substitute strategies
- Vertical mergers are one way of exercising vertical control (we will not focus on the ‘make’ or ‘buy’ decisions in this lecture)

- Vertical control differs from horizontal control in that an input can be seen as a product complement to
  - Downstream product, e.g. car production and retail sale of car
  - Other (non-substitute) inputs; e.g. chassis and engine in a car
- Market power effects much less likely than for horizontal control
  - There can be however foreclosure effects (we will come to that in a bit)
- Efficiency effects
  - Vertical mergers typically create more efficiencies and less market power than horizontal mergers
  - Efficiencies include timely coordination of production and distribution, specific investment, joint development of innovation, etc.
  - Vertical mergers can be framed as eliminating externalities

- Vertical relationships result in three 'externalities' in the sense that independent firms at one stage of production do not take account of the consequences for firms at another stage of production
  - Double marginalisation $\rightarrow$ prices too high
  - Downstream moral hazard $\rightarrow$ service quality too low
  - Downstream free riding $\rightarrow$ service quality too low
- 'Chicago view': vertical mergers can only be beneficial
  - Merger can internalise these externalities
  - Cannot 'leverage' market power from one stage of production to the next (i.e. completely opposite to horizontal mergers)
- 'Post Chicago view': vertical mergers are much less likely to be harmful than horizontal mergers, but can still be harmful
  - Excluding entrants (= full foreclosure)
  - Raising rivals' costs (= partial foreclosure)
  - Facilitating the full exploitation of market power

# Vertical Mergers
## Double Marginalization

- Double marginalization
  - Two independent firms, upstream and downstream, that *each* have market power
  - Each firm then prices at a mark-up over marginal cost
  - Upstream margin enters downstream marginal cost, so prices higher than for joint profit maximization
  - Deadweight loss of pricing above marginal cost is done twice




- Vertical merger results in
  - Elimination of one deadweight loss
  - Higher joint profits
  - Lower final prices
- Note that same result could be achieved from two-part tariffs
  - Part 1 Maximize value created: wholesaler can set the wholesale price at marginal cost which will maximize the total profit for the two
  - Part 2 Use the fixed fee to capture value: use a franchise fee to capture this additional value

- Linear example
    - Two independent monopolists – upstream wholesaler and downstream retailer
    - Let the upstream monopolist marginal cost = $c$ and the wholesale price she sets (under monopoly pricing rule) be $p_w$
    - Let the retailers only marginal cost be $p_w$, and the retail price she sets (under monopoly pricing) be $p_r$
    - Let the downstream demand be given by $q(p) = 1 - p$
- What are the prices and profits as independent firms and under an integrated firm (i.e. under a merger)?

- Independent firms
  - Retailer takes $mc = p_w$ as given and then solves her profit maximizing price
  - Retailer's problem: $\max_{p_r}(p_r - p_w)q(p_r) = \max_{p_r}(p_r - p_w)(1 - p_r)$
  - FOC gives $p_r^* = (1 + p_w)/2$

  - Upstream derived demand: $q(p_w) = 1 - p_r = (1 - p_w)/2$
  - Wholesalers problem: $\max_{p_w}(p_w - c)q(p_w) = \max_{p_w}(p_w - c)(1 - p_w)/2$
  - FOC gives $p_w^* = (1 + c)/2$

  - Substitute $p_w^*$ into $p_r^*$, solve for prices (for consumers), quantities and sum of profits in terms of exogenous parameters

$$p_r^* = \frac{3+c}{4} \qquad q_r^* = \frac{1-c}{4}$$

$$\pi_w + \pi_r = [\frac{(1-c)^2}{8}] + [\frac{(1-c)^2}{16}] = \frac{3(1-c)^2}{16}$$

- Under an integrated firm
  - Integrated firm's problem: $\max_{p_I}(p_I - c)q(p_I) = \max_{p_I}(p_I - c)(1 - p_I)$
  - FOC gives: $p_I^* = (1 + c)/2$
  - Substitution into demand and profit function give

$$p_I^* = \frac{1+c}{2} < p_r \qquad q_I^* = \frac{(1-c}{2} > q_r$$
$$\pi_I = \frac{(1-c)^2}{4} > \pi_w + \pi_r$$

- Thus under integration **prices are lower, profits are higher and consumer welfare is higher**

- Downstream firms need to provide some additional value added services (for instance promotional efforts) which effect the profits of both upstream and downstream firms
- Since there is a cost to such services, the retailer would only offer that level of services where the marginal cost equals its own marginal benefit and would ignore any benefit of additional services that would go to the wholesaler
- End result would be under provision of services
- An integrated firm would not ignore this additional marginal benefit
- Example ...

- Example ...
  - Let $s$ be the level of services provided by retailer (e.g. pre-sale information) and let $\sigma(s)$ be the cost of this service and $\sigma'(s) = d(\sigma)/ds > 0$
  - Let demand be a function of both price and service $q(p, s)$
  - Integrated monopolist's problem:

$$\max_s \pi_I = (p - c)q(p, s) - \sigma(s)$$

$$\text{FOC:} \quad \frac{\partial \pi_I}{\partial s} = (p - c)\frac{\partial q}{\partial s} - \sigma'(s) = 0$$

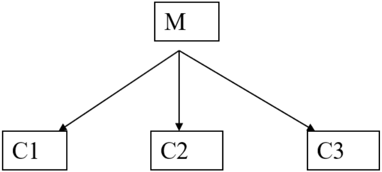
  - Independent retailer's problem:

$$\max_s \pi_I = (p - p_w)q(p, s) - \sigma(s)$$

$$\text{FOC:} \quad \frac{\partial \pi_r}{\partial s} = (p - p_w)\frac{\partial q}{\partial s} - \sigma'(s) = 0$$

  - Note that $(p - c) > (p - p_w)$ which implies $s_I^* > s_r^*$ – this is because the retailer ignores the additional profit margin $(p_w - c)\partial q/\partial s$

- Intra-brand competition
- This externality applies when there are multiple retailers
  - Consumers get information from retailer with highest $s$
  - Then buy from retailer with lowest $p$
  - Implies retailers will provide $s = 0$
  - eg visit bricks and mortar shop to learn about differences in products and then buy from the cheapest internet site

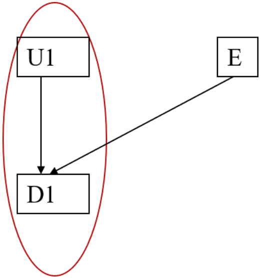



- All three externalities can be internalised/eliminated by vertical merger

- Monopoly profits can only be taken once
- Assume no threat of upstream entry
- In absence of the above externalities, downstream price is unaffected by integration
- In presence of the above externalities, VI results in a better outcome for both consumers and firms
  - Chicago view is that neither VI nor VRs extend market power, though they may be good for efficiency
  - Policy should always allow vertical mergers

- Chicago School shows that, for a completely unchallenged monopolist, monopoly profits can only be taken once. But argument can fail if that does not hold
  - Entry can be 'foreclosed' (or exit can be forced)
  - Rivals can be disadvantaged ('raising rivals costs') (i.e. 'Partial foreclosure')
- Other reservations
  - Price discrimination can be facilitated; i.e. enhanced ability to exploit existing market power
  - Collusion can be facilitated; Sharing price information with horizontal rivals
  - Unfair information advantage in bidding markets; e.g. Sky/Manchester United and Premier League TV rights bidding
  - Unfair information advantage in innovation markets; e.g. Navteq/Nokia

- Foreclosure of potential entrant: vertical merger reduces marketing and input supply options for entrants
- Upstream entrant finds it harder to sell (possibly innovative product)

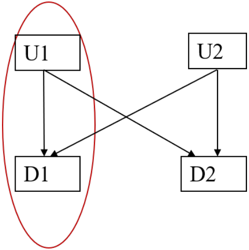



- Similar possibility of downstream foreclosure (if refuse to sell crucial input, or raise its price)

- What if U1 and D1 merge?

- Raising rival's costs: partial or full foreclosure
  - Partial foreclosure if rival's costs are raised, so becomes a less effective competitor
  - Full foreclosure if rival exits

- Downstream foreclosure if U1 raises prices to D2 (or refuses to deal, so U2 becomes de facto monopolist)




- Upstream foreclosure if D1 reduces purchases from U2, so U2 loses scale economies (or D2 gains market power)

**Appendix – Example with Linear Demand System**

(1) How to back out marginal costs
(2) Compute new prices under a merger
(3) Allow cost efficiency for merging firms

- Suppose demand functions are linear, and the demand for $j$th product is given by

$$q_j = a_j + \sum_{k=1}^{J} b_{jk} p_j$$

and marginal cost for each product is $mc_j$

- We can write the demand equation in matrix notation as

$$\mathbf{q} = \mathbf{a} + \mathbf{B}\mathbf{p}$$

where for instance vector $\mathbf{a}$ and matrix $\mathbf{B}$ are given by

$$\mathbf{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_j \\ \vdots \\ a_J \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & \dots & b_{1k} & \dots & b_{1J} \\ \vdots & & \vdots & & \vdots \\ b_{j1} & \dots & b_{jk} & \dots & b_{jJ} \\ \vdots & & \vdots & & \vdots \\ b_{J1} & \dots & b_{Jk} & \dots & b_{JJ} \end{bmatrix}$$

- Suppose there are 6 independent firms and 6 products
  - Demand functions are linear and previously estimated to be

$$q_j = 10 - 2p_j + 0.3 \sum_{k \neq j}^{5} q_k$$

  - In a typical market, say price and quantity are **observed** to be 4.8 and 7.6 respectively for all the products
  - $\mathbf{p}' = (4.8, 4.8, 4.8, 4.8, 4.8, 4.8)$ and $\mathbf{q}' = (7.6, 7.6, 7.6, 7.6, 7.6, 7.6)$
- Using the equations above we can back out the marginal cost and compute markups and price-cost margins

$$\mathbf{B}' = \begin{bmatrix} -2 & .3 & .3 & .3 & .3 & .3 \\ .3 & -2 & .3 & .3 & .3 & .3 \\ .3 & .3 & -2 & .3 & .3 & .3 \\ .3 & .3 & .3 & -2 & .3 & .3 \\ .3 & .3 & .3 & .3 & -2 & .3 \\ .3 & .3 & .3 & .3 & .3 & -2 \end{bmatrix} \qquad \mathbf{\Theta} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Let $\mathbf{a}$ be column vector of intercept terms (all equal to 10 in this example), so $\mathbf{a}' = (10, 10, 10, 10, 10, 10)$
- Then from $\mathbf{p} = \mathbf{c} + \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$ and $\mathbf{\Omega} = -\mathbf{\Theta} \cdot \mathbf{B}'$, it follows that estimated marginal cost $\widehat{\mathbf{c}}$ can be computed as

$$\mathbf{c} = \mathbf{p} \quad - \quad \mathbf{\Omega}^{-1} \quad \mathbf{q}(\mathbf{p})$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

- Thus we have backed out the marginal costs (all equal to 1 in this example) with *price cost margins* being 100(4.8-1)/4.8 = 79.16% for each product

- Equipped with marginal costs and demand parameters, we can now simulate new equilibrium prices and quantities
- For the moment, let's continue with our linear demand system
- We start by determining/solving for Nash-equilibrium given the set of $J$ demand equations $\mathbf{q} = \mathbf{a} + \mathbf{B}\mathbf{p}$ and the set of $J$ price equations $\mathbf{p} = \mathbf{c} + \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$ derived from the first order conditions specific to this linear demand system

- The set of 2J equations $\mathbf{q} = \mathbf{a} + \mathbf{B}\mathbf{p}$ and $\mathbf{p} = \mathbf{c} + \mathbf{\Omega}^{-1}\mathbf{q}(\mathbf{p})$ jointly determine equilibrium price and quantity vectors in any market
  - Write the 2 matrix form equations as

$$\mathbf{q} = \mathbf{a} + \mathbf{B}\mathbf{p} \qquad \text{and} \qquad \mathbf{q} = \mathbf{\Theta} \cdot \mathbf{B}'(\mathbf{p} - \mathbf{c})$$

  - They can be stacked with the endogenous variables p,q on the LHS as

$$\begin{bmatrix} (\mathbf{\Theta} \cdot \mathbf{B}') & \mathbf{I} \\ -\mathbf{B} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} (\mathbf{\Theta} \cdot \mathbf{B}') & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{a} \end{bmatrix}$$

  where $\mathbf{I}$ are $\mathbf{0}$ are $J \times J$ identity and zero matrices respectively, and hence

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} (\mathbf{\Theta} \cdot \mathbf{B}') & \mathbf{I} \\ -\mathbf{B} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} (\mathbf{\Theta} \cdot \mathbf{B}') & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{a} \end{bmatrix}$$

- The set of equations

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{\Theta} \cdot \mathbf{B}') & \mathbf{I} \\ -\mathbf{B} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} (\boldsymbol{\Theta} \cdot \mathbf{B}') & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{a} \end{bmatrix}$$

  can be easily solved using any matrix based software (Matlab, R, Mathematica, SAS, STAT, etc. ... and can even be programmed in Excel)

- Thus given the demand parameters of a linear demand system, marginal costs and the ownership matrix, we get a unique Nash equilibrium solution in prices and quantities
  - Let $\boldsymbol{\Theta}$ and $\mathbf{B}$ be as specified for the linear demand system for six products owned by six separate firms, and let $c' = (1, 1, 1, 1, 1, 1)$
  - Then

$$\mathbf{p}^* = \begin{bmatrix} 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \\ 4.8 \end{bmatrix} \qquad \mathbf{q}^* = \begin{bmatrix} 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \\ 7.6 \end{bmatrix}$$

- Suppose firms 1 and 2 merge, firms 3 and 4 merge and firms 5 and 6 merge – then all we need to do is change the owenership matrix $\mathbf{\Theta}$ to reflect the new post merger ownership and resolve the system of equations using the new ownership matrix

- Let the pre merger and post merger ownership matrices be given by $\mathbf{\Theta}_0$ and $\mathbf{\Theta}_1$ respectively (i.e., for time 0 and 1)

$$\mathbf{\Theta}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrowtail \mathbf{\Theta}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- Now solve for $\mathbf{p}$ and $\mathbf{q}$ using $\mathbf{\Theta}_1$

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} (\mathbf{\Theta}_1 \cdot \mathbf{B}') & \mathbf{I} \\ -\mathbf{B} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} (\mathbf{\Theta}_1 \cdot \mathbf{B}') & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{a} \end{bmatrix}$$

- Suppose firms 1 and 2 merge, firms 3 and 4 merge and firms 5 and 6 merge
- The old and new equilibria are as follows

| | **Pre-merger values** | | | | **Post-merger values** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Product | p | q | (p-c)/p | $\pi$ | p | q | (p-c)/p | $\pi$ | $\% \Delta p$ |
| 1 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |
| 2 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |
| 3 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |
| 4 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |
| 5 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |
| 6 | 4.8 | 7.6 | 79.2% | 28.88 | 5.32 | 7.34 | 81.2% | 31.70 | 10.80% |

- Overall prices increase by 10.8% for each product and total output falls, which would reduce consumer surplus
- What if there was an efficiency defence – say 25% reduction in costs?

- Suppose there is a merger specific efficiency defence – that marginal costs would reduce by 25% – then in addition to changing the ownership matrix, we can multiply mc by 0.75 and resolve

- Let the pre merger and post merger ownership matrices be given by $\boldsymbol{\Theta}_0$ and $\boldsymbol{\Theta}_1$ respectively (i.e., for time 0 and 1)

$$\boldsymbol{\Theta}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \rightarrowtail \quad \boldsymbol{\Theta}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- Now solve for $\mathbf{p}$ and $\mathbf{q}$ using $\boldsymbol{\Theta}_1$

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{\Theta}_1 \cdot \mathbf{B}') & \mathbf{I} \\ -\mathbf{B} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} (\boldsymbol{\Theta}_1 \cdot \mathbf{B}') & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0.75 \cdot c} \\ \mathbf{a} \end{bmatrix}$$

- Suppose firms 1 and 2 merge, firms 3 and 4 merge and firms 5 and 6 merge and costs reduce by 25% due to mergers

- The the old and new equilibria are as follows

| | **Pre-merger values** | | | | **Post-merger values** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Product | p | q | (p-c)/p | $\pi$ | p | q | (p-c)/p | $\pi$ | % $\Delta p$ |
| 1 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |
| 2 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |
| 3 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |
| 4 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |
| 5 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |
| 6 | 4.8 | 7.6 | 79.2% | 28.88 | 5.13 | 7.44 | 85.4% | 32.54 | 6.77% |

- Overall prices still increase by 6.77% and output is reduced so merger does not improve consumer surplus
- Can also compute change in total profits and compare to the change in total CS for welfare criteria

- Thus, we can modify the ownership matrix and/or the vector of estimated (or known) marginal costs to simulate unilateral effects
- In the previous analysis, the demand curves were linear and hence the solutions, the Nash-Bertrand equilibrium, was easy to compute no matter how large the system of equations (dictated by $J$)
- More generally, the most appropriate demand system may not be linear but the overall process stays the same

# REFERENCES


- Davis, P. and Garcés, E. (2010). “Merger Simulations” Chapter 8 in *Quantitative techniques for competition and antitrust analysis*. Princeton University Press.
- Farrell, J. and Shapiro, C. (1990). Horizontal mergers: An equilibrium analysis. American Economic Review, 80(1):107–126.
- Farrell, J. and Shapiro, C. (2010). Antitrust evaluation of horizontal mergers: An economic alternative to market definition. B.E. Journal of Theoretical Economics; Policies and Perspectives, 10(1): 1-39.