Product Differentiation

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- When firms sell identical (homogenous) products, price competition leads consumers to buy from the cheapest seller
  - Prices converge to cost
  - Even with just two sellers, prices go from monopoly to competitive level (Bertrand's paradox)
  - Realistic? In fact consumers may buy from some more expensive providers as well
- Most markets consists of products that are close but not perfect substitutes of each other
  - Starbucks coffee same as a cup of joe from a pit stop on a highway?
  - Product may differ in their locational or physical characteristics
  - Or they may differ in the mind of the customer due to branding/advertising – Coke vs Pepsi, branded vs generic drugs

- Consumer heterogeneity
  - If products are not identical, consumers may have preference over them
  - Some like Coke, others prefer 7up
  - Some like both at different times (taste for variety)
  - Some willing to pay more for a particular characteristic
- Implications of product differentiation
  - Why do firms differentiate their products?
  - How do they differentiate them?
  - What set of products constitute a market?
  - What are the consequences for consumers?
  - How should we model price and quantity competition?

Why do firms differentiate their products?

- What are the tradeoffs? *Market power* versus *market size*
  - If you are very similar, it is easy for others to steal your business
  - No brand loyalty or consumer inertia – small deviation in price and consumers may abandon you
  - But you have access to a larger market
  - If you differentiate your product, some will abandon your competitor because they prefer what you have to offer (7up to those that don't like Coke as much)
  - Harder to steal your customers – you can enjoy some market power
  - But you have access to a smaller market

How do they differentiate them?

- Products can be differentiated *horizontally* – due to differences in taste across consumers – or *vertically* – due to how much value consumers attach to quality differences
  - Some like 7up while others like Coke due to differences in tastes, but while everyone likes a faster computer, some get more utility from a faster computer
- **Horizontal differentiation**: more a matter of specialization of products to a specific subclass of consumers, in a way that does not mean that one of the types of products offered is considered better by all consumers
  - Design: Alessi vs Bodum. Same type of prices, but very different esthetics (colorful vs sober); Drugs: Adderall vs Concerta for ADHD - both expensive and high quality, but different segments (different molecules that inhibit the reuptake or release of dopamine in different ways to help symptoms)
- **Vertical differentiation**: Hierarchical quality dimension, so that everyone agrees on the ranking of products offered on the market
  - BMW vs Opel; Adderall XR vs Adderall (once a day vs multiple times a day drug for ADHD)?

A simple thought experiment to understand the difference between vertical and horizontal differences in goods

- Imagine two goods that have the same price
  - if different consumers prefer different goods, then they are horizontally differentiated
  - if all consumers prefer the same good, then they are vertically differentiated (however, what if the cost to produce is different due to quality differences – hence modify to: when all consumers prefer the same product when both are priced at marginal cost)
- In reality, it may be not be always as clear as above if products are horizontally or vertically differentiated

What set of products constitute a market?

- Market definition
  - All alcoholic drinks, or beers and ales vs spirits vs wines etc. separately?
  - Therapeutic class or class separated by molecules?

What are the consequences for consumers?

- People like choices ... increases utility
- May cost may more to provide additional variants ...
- Product differentiation may also reduce price competition ...
- Are there two many or two few choices?

How should we model price and quantity competition?

- Agenda for today
- Learn about various ways of modelling industries with differentiated products
- These help us understand/answer some of the issues listed above

Horizonal Differentiation

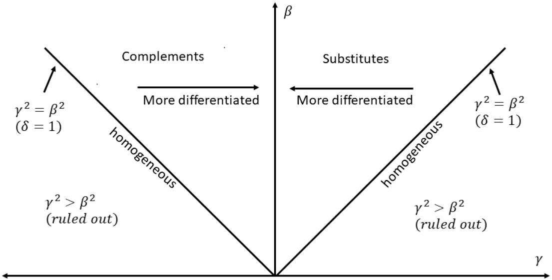
- Differentiated duopoly w./ Representative consumer
  - General model setup
  - Competition under quantity (Cournout)
  - Competition under prices (Bertrand)
- Hotelling Model w./ Heterogenous consumer
  - General model setup
  - Product differentiation with exogenous price (choose location)
  - Product differentiation with endogenous price (choose location and price)
  - Entry and Salop circle (given in slides but may/will skip)
- Vertical differentiation
  - General model setup
  - Product differentiation with endogenous price (choose location and price)

Differentiated Duopoly
Representative Consumer Model

- Two firms producing with imperfect substitutes
- Inverse demand functions given by

$$p_1 = \alpha - \beta q_1 - \gamma q_2 \qquad p_2 = \alpha - \gamma q_1 - \beta q_2$$

where $\beta > 0, \quad \beta^2 > \gamma^2$



- $\beta^2 > \gamma^2$ implies that own price effect dominates the cross-price effect
- Let $\delta = \gamma^2/\beta^2$; then $\delta > 1$ not allowed, $\delta = 1$ they are homogenous, $\delta < 1$ they are differentiated, and $\delta = 0$ unrelated products (or highly differentiated)

- The implied demand functions are

$$q_1 = a - bp_1 + cp_2 \qquad q_2 = a + cp_1 - bp_2$$
$$\text{where } a = \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}, \quad b = \frac{\beta}{\beta^2 - \gamma^2}, \quad c = \frac{\gamma}{\beta^2 - \gamma^2}$$

- How do the equilibrium outcomes differ from the homogenous case when firms compte in
  - prices (Bertrand)
  - outputs (Courtnout)
  - sequential price setting (Stackelberg)?

  - Recall: In monopoly or perfect competition, the equilibrium outcomes depend only on preferences, demand, supply, technology; In oligopolistic competition, equilibrium outcomes also depends on the assumptions about how a firm anticipates its competitors will react to its own actions

BERTRAND COMPETITION
STRATEGIC COMPLEMENTS

- Firm costs are $C(q_i) = k_i q_i$ (can set $k_i = 0$, general results still hold)

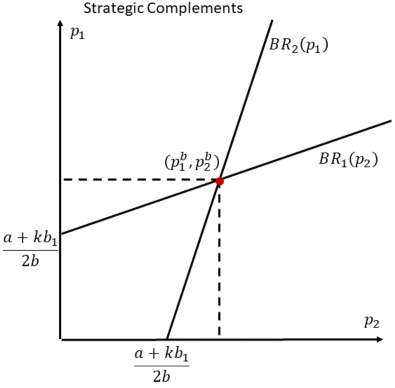
$$\max_{p_i} \pi_i = (p_i - k_i)(a - bp_i + cp_j)$$

- FOC $(\partial \pi_i / \partial p_i = 0)$ give $a - 2bp_i + cp_j + k_i b = 0$

- Firms best-response functions are:

$$BR_1(p_2) = \frac{a + k_1 b + cp_2}{2b}$$
$$BR_2(p_1) = \frac{a + k_2 b + cp_1}{2b}$$

- **Upward sloping BR functions (strategic complements)**
- Equilibrium where they cut each other
- How would the curves shift if cost (or other parameters) change?



- Nash Bertrand Equilibrium (with $k_i = 0$ marginal cost)

$$p_i^b = \frac{a}{2b-c} = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma}$$
$$q_i^b = \frac{ab}{2b-c}$$
$$\pi_i^b = \frac{a^2 b}{(2b-c)^2} = \frac{\alpha^2\beta(\beta-\gamma)}{(2\beta-\gamma)^2(\beta+\gamma)}$$

- Note the following
  - Recall in homogenous case with Bertrand competition, $p = 0$ and $\pi = 0$ (assuming marginal cost is zero)
  - If $\beta = \gamma$ (i.e. homogenous products) then we get same results above
  - Product differentiation leads to prices above marginal cost

- **Proposition:** *In Bertrand game with differentiated products, profits increase as degree of differentiation increases*
- Firms have an incentive to differentiate to avoid toughness of competition

- Firm costs are $C(q_i) = k_i q_i$ (can set $k_i = 0$, general results still hold)

$$\max_{q_i} \pi_i = (\alpha - \beta q_i - \gamma q_j) q_i - k_i q_i$$

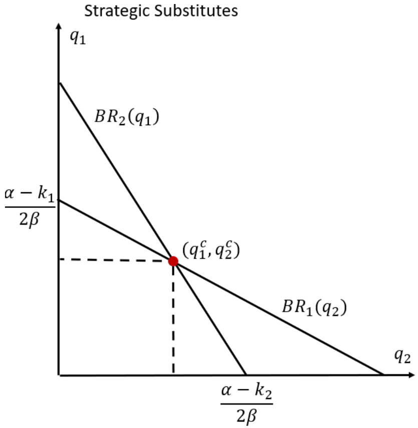
- FOC ($\partial \pi_i / \partial q_i = 0$) give $\alpha - 2\beta q_i - \gamma q_j - k_i = 0$

- Firms best-response functions are:

$$BR_1(q_2) = \frac{\alpha - \gamma q_2 - k_1}{2\beta}$$

$$BR_2(q_1) = \frac{\alpha - \gamma q_1 - k_2}{2\beta}$$

- **Downward sloping BR functions (strategic substitutes)**
- Equilibrium where they cut each other
- How would the curves shift if cost (or other parameters) change?



- Nash Cournout Equilibrium (with $k_i = 0$ marginal cost)

$$p_i^c = \frac{\alpha\beta}{2\beta + \gamma}$$
$$q_i^c = \frac{\alpha}{2\beta + \gamma}$$
$$\pi_i^b = \frac{\alpha^2\beta}{(2\beta + \gamma)^2}$$

- Note the following
  - If $\gamma = \beta$ we get the homogenous Cournot model
  - As γ approaches β (from below), best response functions become steeper profit maximizing output more sensitive to output of the other firm
  - As γ approaches 0, best response functions become constant (zero slopped)

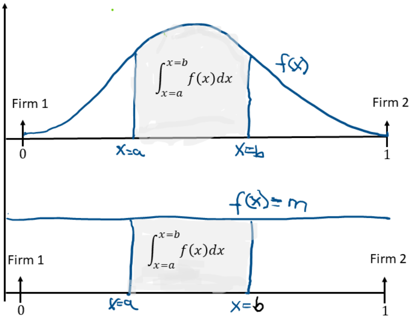
- **Proposition.** *In Cournot game with differentiated products, profits increase as degree of differentiation increases*
- Firms have an incentive to differentiate to avoid toughness of competition

- **Proposition.** *In a differentiated products industry*
  - *Prices under Cournot competition are higher than under Bertrand competition*
  - *The more differentiated the products, the smaller the difference between Cournot and Bertand prices*
  - *In the limit that the products are independent, no difference in the prices under the two models*

- Recall the standard Stackelberg model (homogenous products, sequential move, firms compete in quantity)
  - Compared to the standard Cournout simultaneous move model, prices in the Stackelberg model are lower, and combined outputs is higher
  - However first mover output is higher and second mover output is lower compared to the standard Cournout game
  - Also profits of first mover are greater than the simultaneous Cournout game and greater than the second mover's profits (where the latter are lower than the simultaneous Cournout game)
- By comparison, under a sequential-movers price game (Bertrand)
  - Both firms make higher profit relative to the static differentiated products Bertrand game with simultaneous moves
  - However the first mover gains less relative to the second mover when comparing the gain over the static differentiated products Bertrand game with simultaneous moves, i.e., $\pi_1^s - \pi_1^b < \pi_2^s - \pi_2^b$
  - The first mover makes lower profit than the second mover in a price setting game (generally where strategies are complements)

Hotelling Model
Heterogenous Consumer Model

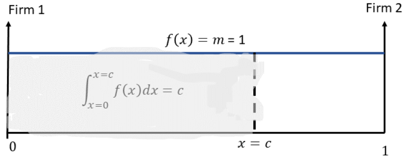
- Exactly two sellers (selling ice creams, soft drinks, widgets, whatever ...) that are identical in every respect except for their location
- Let the length of the linear city be one unit and suppose that each firm is located at its far ends. Thus firms are located at location 0 and 1 (if they were physically 5,012 feet apart, you could normalize that to be 1 unit)



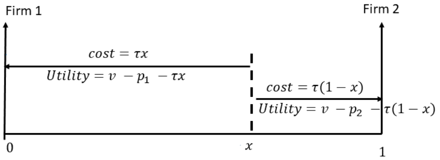
Total population between x=a and x=b is given by the shaded area

- Infinitely many consumers all living along the line (a continuum) at various points x between 0-1.
- Hypothetical density of consumers is described in the picture (two examples: symmetric bell shaped around the city center or uniform)
- We will typically assume uniform density (lower picture)
- Normalize the total population to 1 (set m equal to distance between firms, $m = 1$ or unit mass).

- Consumer Heterogeneity
  - We describe a consumer by the location (address) on the line
  - Consumer living at $x = c$ is c units away from firm 1 and $(1 - c)$ away from firm 2
  - Number of consumers (or fraction before normalizing) of consumers to the left of consumer at $x = c$ is given by the shaded area to the left of $x = c$
  - In uniform distribution with mass 1, that is just c
  - Similarly, number of consumers living to the right of $x = c$ are given by $(1 - c)$



- Suppose there is a travel cost of $\$\tau$ per unit of distance to visit a store



  - For a person located at x, what is the cost to visit store 1 and store 2?

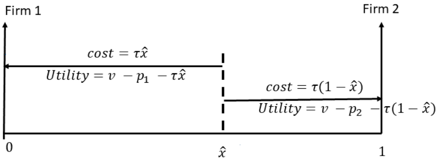
    τx and $\tau(1-x)$ respectively

  - If the implicit utility of consuming 1 unit of a product from either store is v, and each store charges price p_1 and p_2 respectively, then the total utility from consumption from either store is given by

$$u_1 = v - p_1 - \tau x \qquad \text{and} \qquad u_2 = v - p_2 - \tau(1-x)$$

  - The travel cost τ is either a physical cost of travelling to a store or the disutility you get from purchasing a product that is less than your ideal product characteristic

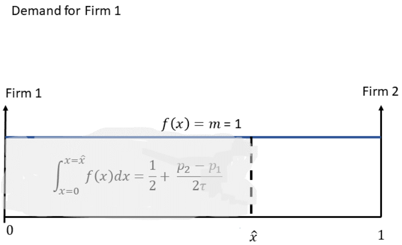
- As long as prices are not very high so that everyone buys from firm 1 or firm 2, there is some consumer at location $\widehat{x}$ who will be indifferent in going to store 1 or store 2

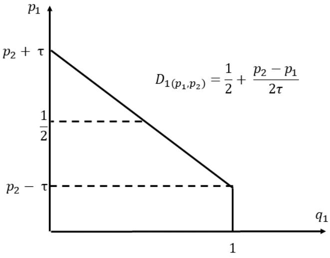


- Identity of the indifferent consumer
  - $u_1 = v - p_1 - \tau\widehat{x} = v - p_2 - \tau(1 - \widehat{x}) = u_2$
  - $\widehat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$
  - If prices are identical, then the indifferent consumer is located midway

Hotelling Model
General Setup

- Suppose that v is high enough so that utility from buying from firm 1 or firm 2 is positive given the values of p_i and τ (ie. market is covered) and that each consumer buys a max of only one unit
- How do we find the **demand** faced by firm 1 (and firm 2)?
  - Identity of the indifferent consumer: $\widehat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$
  - Consumers left of $\widehat{x}$ buy from firm 1, while consumers right of $\widehat{x}$ buy from firm 2
  - Given that the marked is covered, count (integrate) all consumers that buy from the two firms
  - $D_1(p_1, p_2) = \int_0^{\widehat{x}} f(x)dx = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$





- We will use this general setup to see how firms may choose location – i.e., how much to differentiate, when
  - prices are fixed
  - prices and location are both selected by firms
- This type of model is also useful for understanding vertical differentiation

- Horizontal differentiation with fixed price and endogenous location
  - Key idea: model the fact that product specifications and design are part of the choice variables of the firm (i.e. of its strategy set)
  - For now, to keep things very simple, assume that firms *only* choose specification and not price
  - Prices are constant, fixed at some exogenous level $\overline{p}$. It could be because the market is regulated – consider for example the case of many professions where this is the case
  - Consumers uniformly distributed on [0,1] and with mass 1 – a consumer's location is his ideal point in the product space
  - Consumers buy up up to one unit of the good (unit demand)

- Horizontal differentiation with fixed price and endogenous location
  - Consumers incur a linear transportation cost $\tau |x - l_i|$ if they are located at x and consume a good located at l_i
  - Consumer x derives utility:

$$u_i(x) = v - \tau |x - l_i| - \overline{p}$$

    from consuming a product located in l_i at price $\overline{p}$
  - $\tau |x - l_i|$ represents the opportunity cost for the consumer of not consuming her ideal variety
  - So τ measures the substitutability between goods i and j
  - v measures the value of the ideal good, and we shall assume it large so that all consumers will purchase

- Horizontal differentiation with fixed price and endogenous location
  - Firms have to decide to which consumers to tailor their product – in the model this is equivalent to each firm i deciding where to locate on the line, and consumers then decide from whom to buy
  - The marginal cost of producing the good is $c < \overline{p}$
  - Timing of the game:
    - 1) firms simultaneously decide where to locate on [0,1]
    - 2) consumers decide from which firm to buy (if any)

We obtain the following main result:

- **Proposition**: *In the above defined two-firms simultaneous location game with fixed prices, in the unique Nash Equilibrium both firms locate at* $\frac{1}{2}$.
  - They can, but choose not to differentiate their product!

- Horizontal differentiation with fixed price and endogenous location
  - **Step 0 (Preliminary observations)** For any two locations $l_1 < l_2$, there is a consumer type $\widehat{x}$ who is indifferent between the two offers
  - This consumer is located in the center of the segment $[l_1, l_2]$ – i.e.

$$\widehat{x} = \frac{l_1 + l_2}{2}$$

  - All consumers to the left of $\widehat{x}$ prefer to buy from firm 1 and all consumers to the right prefer to buy from firm 2
  - It follows that the demand faced by firm 1 is

$$Q_1(l_1, l_2) = \frac{l_1 + l_2}{2}$$

while the demand faced by firm 2 is given by:

$$Q_2(l_1, l_2) = 1 - \left(\frac{l_1 + l_2}{2}\right)$$

- Horizontal differentiation with fixed price and endogenous location
  - Firms maximize their profits w.r.t. product location given the location picked by competitor
  - Profits of firm i are given by:

$$\pi_i(l_i, l_j) = \begin{cases} (\overline{p} - c)\,\frac{l_i + l_j}{2} & \text{if } l_i < l_j \\ (\overline{p} - c)\,\frac{1}{2} & \text{if } l_i = l_j \\ (\overline{p} - c)\left(1 - \frac{l_i + l_j}{2}\right) & \text{if } l_i > l_j \end{cases}$$

- Horizontal differentiation with fixed price and endogenous location
  - **Step 1** *The two firms must pick the same location, otherwise there is a profitable unilateral deviation.*
  - Suppose that locations are different, with $l_1 < l_2$
  - Then firm 2 can gain by moving to $\widetilde{l_2} = l_1 + \varepsilon$ for ε very small
  - Indeed, by doing so, it increases its demand from

$$1 - \underbrace{\frac{l_1 + l_2}{2}}_{\text{indifferent consumer type}}$$

to approximately

$$1 - l_1$$

which is clearly larger than $1 - \frac{l_1+l_2}{2}$ given $l_1 < l_2$

- Horizontal differentiation with fixed price and endogenous location
  - **Step 2** *If* $l_1 = l_2 = l^* \neq \frac{1}{2}$, *there is a profitable unilateral deviation*
  - Suppose for example that the location is below $\frac{1}{2}$, i.e. $l^* < \frac{1}{2}$
  - Each firm obtains a demand of $\frac{1}{2}$
  - If for example firm 2 moves to

    $$l^* + \varepsilon,$$

    for ε very small, it now obtains demand roughly

    $$1 - l^*$$

    which is clearly strictly larger than $\frac{1}{2}$ given that $l^* < \frac{1}{2}$

- Horizontal differentiation with fixed price and endogenous location
  - **Step 3** *If* $l_1 = l_2 = \frac{1}{2}$, *there is no profitable unilateral deviation*
  - Here, any firm that deviates serves less than half the market and thus makes less profits than by sticking to the location $\frac{1}{2}$
  - If firm i deviates to $\widetilde{l}_i > \frac{1}{2}$, it obtains:

$$1 - \Bigg(\underbrace{\frac{\frac{1}{2} + l_1}{2}}_{\text{indifferent consumer type}} \Bigg) < \frac{1}{2}$$

  - If instead firm i deviates to $\widetilde{l}_i < \frac{1}{2}$, it obtains:

$$\frac{l_1 + \frac{1}{2}}{2} < \frac{1}{2}$$

  - It follows that this profile of locations is an equilibrium ■

- Horizontal differentiation with fixed price and endogenous location

  Comments (1):

  - So firms could differentiate their products but choose not to. So the model does not provide an explanation of price differentiation
  - Individual maximization of market shares leads firms to pick the same central location. That's where they can best reach most consumers (this is reminiscent of the median voter theorem in downsian voting models)

- Horizontal differentiation with fixed price and endogenous location

  Comments (2):

  - A monopolist might very well decide to serve from both locations $\frac{1}{4}$ and $\frac{3}{4}$, so that a monopolist may be better for welfare
  - In other words, the introduction of two firms and competition here is not welfare improving, given firms' identical location choice
  - It can be shown that the result is robust to the transport cost function (for example quadratic instead of linear) but it is not robust to the number of firms – it does not hold for example if there are three firms

  A very unrealistic assumption is fixed prices. One ought to introduce endogenous prices and see whether the result still holds under this assumption

- Horizontal differentiation with endogenous location and endogenous price
  - Suppose now that firms choose prices in addition to location
  - Firm i now chooses location l_i and its price p_i
  - Assume (for analytical tractability) a quadratic transportation cost, i.e.

$$TC(x, l_i) = \tau\,(x - l_i)^2$$

  - Consumers are uniformly distributed on [0,1]
  - Firms have an identical marginal cost of production c

- Horizontal differentiation with endogenous location and endogenous price
  - Timing of the game:
    - 1) Firms simultaneously choose location
    - 2) Firms simultaneously set prices
    - 3) Consumers decide from whom to buy (if at all)
  - We solve the game by backwards induction, i.e.:
    - first solve for prices given location
    - then solve for optimal location picked by firms given prices
  - **Proposition:** *In the subgame perfect equilibrium of the two stage location and pricing game with quadratic transport costs, firms choose* $l_1 = 0$ *and* $l_2 = 1$, *i.e. the differentiate maximally.*

- Horizontal differentiation with endogenous location and endogenous price
  - **Step 1** *(optimal prices given locations)*
  - Assume without loss of generality that $l_1 \leq l_2$
  - The indifferent consumer $\widehat{x}$ given l_1, l_2, p_1, p_2 satisfies:

$$v - \tau\left(\widehat{x} - l_1\right)^2 - p_1 = v - \tau\left(\widehat{x} - l_2\right)^2 - p_2$$

i.e.

$$\tau\left(\widehat{x} - l_1\right)^2 + p_1 = \tau\left(\widehat{x} - l_2\right)^2 + p_2$$

  - We can solve the above for $\widehat{x}$ and obtain:

$$\widehat{x}(p_1, p_2, l_1, l_2) = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau(l_2 - l_1)}$$

- Horizontal differentiation with endogenous location and endogenous price
  - Given p_1, p_2, l_1, l_2, the demand faced by the firms is thus given by:

$$\text{Firm 1: } Q_1(p_1, p_2, l_1, l_2) = \widehat{x}(p_1, p_2, l_1, l_2)$$

and

$$\text{Firm 2: } Q_2(p_1, p_2, l_1, l_2) = 1 - \widehat{x}(p_1, p_2, l_1, l_2)$$

  - So the profits functions are:

$$\pi_1(p_1, p_2, l_1, l_2) = (p_1 - c)\,\widehat{x}(p_1, p_2, l_1, l_2)$$

and

$$\pi_2(p_1, p_2, l_1, l_2) = (p_2 - c)\,(1 - \widehat{x}(p_1, p_2, l_1, l_2))$$

- Horizontal differentiation with endogenous location and endogenous price
  - We take FOCs to identify the optimal price for each firm i given l_1, l_2, p_j
  - Solving the obtained system of equations, one obtains the unique price equilibrium for each location pair $l_1 \leq l_2$
  - So we solve

$$\frac{\partial \pi_1}{\partial p_1} = 0, \frac{\partial \pi_2}{\partial p_2} = 0,$$

which yields

$$\begin{aligned} p_1^*(l_1, l_2) &= c + \frac{\tau}{3}(l_2 - l_1)(2 + l_1 + l_2) \\ p_2^*(l_1, l_2) &= c + \frac{\tau}{3}(l_2 - l_1)(4 - l_1 - l_2) \end{aligned}$$

  - Note that the two prices converge to c when the distance $(l_2 - l_1)$ converges to 0 – not surprising, as we thereby converge to homogeneous goods!

- Horizontal differentiation with endogenous location and endogenous price
  - **Step 2** *(Solving for the optimal location given the optimal pricing functions used in the second stage)*
  - We can plug our expressions for $p_1^*(l_1, l_2)$ and $p_2^*(l_1, l_2)$ into the expression for the marginal consumer: $\widehat{x}(p_1, p_2, l_1, l_2)$
  - We thus obtain:

$$\widehat{x}(p_1^*(l_1, l_2), p_2^*(l_1, l_2), l_1, l_2) = \frac{2 + l_1 + l_2}{6}$$

  - Using this expression and plugging this back into the profit functions π_1 and π_2, these are now exclusively functions of the location profile (l_1, l_2) and are given by:

$$\widehat{\pi}_1(l_1, l_2) = \frac{1}{18}\tau(l_2 - l_1)(2 + l_1 + l_2)^2$$

and

$$\widehat{\pi}_2(l_1, l_2) = \frac{1}{18}\tau(l_2 - l_1)(4 - l_1 - l_2)^2$$

- Horizontal differentiation with endogenous location and endogenous price
  - We're now ready to find the optimal location of each firm
  - Note that
    $$\frac{\partial \widehat{\pi}_1(l_1, l_2)}{\partial l_1} < 0 \text{ for all } l_1 \in [0, l_2)$$
    and
    $$\frac{\partial \widehat{\pi}_2(l_1, l_2)}{\partial l_2} > 0 \text{ for all } l_2 \in (l_1, 1]$$
  - So in the firms choose locations $l_1^* = 0$ and $l_2^* = 1$ ■

- Horizontal differentiation with endogenous location and endogenous price

  Comments (1):

  - So spatial competition with endogenous prices leads to **maximal differentiation**
  - This stands in sharp contrast to the locational competition model with fixed prices
  - Clearly, firms choose different locations so as to obtain some monopoly power towards consumers close to them – which allows them to fix high prices for these consumers
  - This force was absent from the first model because it had fixed prices (= impossible to exploit market power by setting higher prices)

- Horizontal differentiation with endogenous location and endogenous price

  Comments (2):

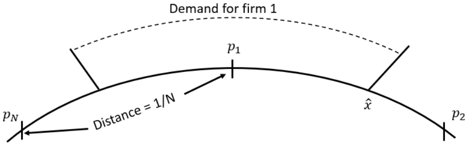
  - So in the model with endogenous price and location, there are always two forces that counteract each other:
    - Force 1: choose different locations so as to enjoy market power (market power effect)
    - Force 2: move towards the middle so as to reach as many consumers as possible (market size effect)
  - In our specification (with quadratic transportation costs), the first effect clearly dominates, but this does not have to be the case

- Horizontal differentiation with endogenous location and endogenous price
  Comments (3):
  - In this model, if we instead allow firms to locate anywhere on the real line, it can be shown that they will locate at $-\frac{1}{4}$ and $\frac{5}{4}$ as opposed to tending towards minus infinity and plus infinity
  - So eventually, the market size effect has some bite
  - In our model, firms choose to be at 0 and 1 – is this too much product differentiation or too little?
    - If a social planner could choose the locations, where would they place the products?
    - The social planner would minimize costs (or maximize utility). ICBS that in that case solution would be $l_1^* = 1/4$ and $l_2^* = 3/4$
  - What if we had used linear transportation costs? Then there is no equilibrium where firms choose both location and prices (one way out is to consider a circular city of unit length)

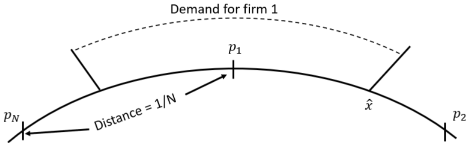
- Monopolistic-competition, free entry and firms choose to enter
- If they enter, they place symmetrically around a circle of unit length 1
  - This model can also be given an interpretation that differs from a physical location interpretation – trains, planes and bus services provide round the clock services, so location on the unit circle can be interpreted as time of departure
- Given entry (with maximal differentiation), firms set prices
- Fixed cost of entry per firm
- Endogenous entry – N firms choose to enter
- **Question:** How many firms will enter, and is this less than or more than socially desirable level (if each firm entry is a distinct brand, are there too many or too few brands?)

- Model set up ...
- Firms choose to enter and locate symmetrically around a unit circle
- If N firms enter, the distance between any two firms is $1/N$
- Fixed cost of entry is F
- Marginal cost is c
- For price p_i and quantity q_i, profit $\pi_i(q_i)$ of firm i is given by

$$\pi_i(q_i) = \begin{cases} (p_i - c)q_i - F & \text{if } q_i > 0 \\ 0 & \text{otherwise} \end{cases}$$



- Consumers are uniformly located around the unit circle
- Each firm competes with its nearest neighbour on the circle (left and right)
- Transportation cost τ is linear
- Demand for firm i
  - if firm 2 and firm N charge a uniform price p (so $p_2 = p_N = p$), then firm 1's demand can be found by locating consumer that are indifferent between purchasing from firm 1 or its two neighbors the left/right
  - the indifferent consumer is located at $\hat{x}$ given by $p_1 + \tau\hat{x} = p\tau(1/N - \hat{x})$



  - Hence

$$\hat{x} = \frac{p - p_1}{2\tau} + \frac{1}{2N} \qquad \text{and} \qquad q_1(p_1, p) = 2\hat{x} = \frac{p - p_1}{\tau} + \frac{1}{N}$$

- Given p, each firm solves the problem

$$\max_{p_i} \pi_i(p_i, p) = p_i q_i - (c q_i + F) = (p_i - c)(\frac{p - p_1}{\tau} + \frac{1}{N}) - F$$

- The FOC give

$$(p - 2p_i + c)/\tau + 1/N = 0$$

- and with symmetric equilibrium where $p_i = p$, we get

$$p_i^* = c + \tau/N$$

- What is N in equilibrium ?

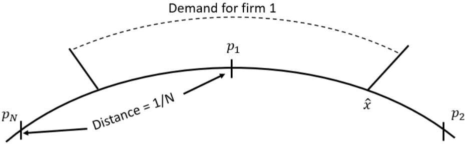
- To find equilibrium N set $\pi_i(p_i^*, p_i^*) = (p_i^* - c)\frac{1}{N} - F = 0$ and solve for N
- Hence

$$N^* = \sqrt{\frac{\tau}{F}}$$
$$p^* = c + \frac{\tau}{N^*} = c + \sqrt{\tau F}$$
$$q^* = \frac{1}{N^*}$$

- Higher transportation costs weakens price competition, increases price, and leads to higher entry
- Costs (skipping algebra ...)
  - cost of entry: $\sqrt{\tau/F}F = \sqrt{\tau F}$
  - transportation costs: $\tau/(4\sqrt{\tau/F}) = \frac{1}{4}\sqrt{\tau F}$
  - total: $\frac{5}{4}\sqrt{\tau F}$
- How does this compare to a social planner's problem (i.e., if we minimize the sum of entry and transportation costs)?

- From the forgoing calculations, a consumer buying from firm 1 is located 0 and $\hat{x}^* = 1/(2N)$ on each side of firm 1 – also, there are $2N$ such intervals, hence the total transportation costs with N firms were

$$T(N) = 2N\tau \int_0^{\frac{1}{2N}} x dx = \frac{\tau}{4N}$$



- A social planner would want to choose N so that the above cost plus the total entry cost $N * F$ is minimized

$$\min_N NF + \frac{\tau}{4N}$$

- The FOC gives

$$N_{sp}^* = \frac{1}{2}\sqrt{\frac{\tau}{F}} = \frac{1}{2}N^*$$

- Thus, under free entry too many firms in equilibrium compared to socially optimal value

$$N_{sp}^* = \frac{1}{2} N^*$$

- Also, costs under social optimum are
  - cost of entry: $\frac{1}{2}\sqrt{\tau/F}F = \frac{1}{2}\sqrt{\tau F}$
  - transportation costs: $\tau/(4\sqrt{\tau/F})/2 = \frac{1}{2}\sqrt{\tau F}$
  - total: $\sqrt{\tau F}$

- Product differentiation resolves the Bertrand paradox
- Firms can differentiate vertically or horizontally
- Models of product differentiation with Cournot, Bertand (and Stackelberg) competition
  - model predictions
  - slope of the best response functions
  - prices higher in Cournot than in Bertand differentiated competition
- Heterogenous consumer models (Hotelling)
  - exogenous prices – minimal differentiation as firms locate at the same point
  - endogenous prices – (quadratic costs) maximal differentiation
  - two effects – differentiate and enjoy marker power, don't differentiate and enjoy larger marker
  - more differentiation than social optimum
- Monopolistic competition circular city – too many firms in equilibrium

Vertical Differentiation

Vertical differentiation

- We now want to consider *strategic* vertical differentiation (i.e. concerning the quality dimension)
- Just as relevant empirically as horizontal differentiation and present on most consumer good markets
- It tends to be assumed that quality differentiation is only a consequence of technology, cost, "know how"
- We want to show that it can also be a strategic/voluntary decision, whereby some firms might for example choose to produce low quality even if high quality is not as such more expensive to produce

VERTICAL DIFFERENTIATION

- We study a model which is a counterpart of the horizontal product differentiation
- Two-stage game:
  - Firms first pick their location (now on a vertical as opposed to horizontal line)
  - and then set prices (i.e. competing in prices)

- The quality of a product is described by a variable s taken from $[\underline{s}, \overline{s}]$
- Consumers agree that high quality is better than low quality, but they are heterogeneous in their valuation of quality
- Each consumer has a preference parameter θ taken from $[\underline{\theta}, \overline{\theta}]$ which measures how much she values quality
- Mass one of consumers distributed uniformly on $[\underline{\theta},\overline{\theta}]$, where $\overline{\theta} > 2\underline{\theta}$

- Given preference type θ, quality s and price p, a consumer obtains the following utility if she consumes:
$$v_i(\theta, s, p) = r + \theta s - p$$
- Note that a consumer values quality s more, the higher is taste parameter θ
- Each consumer buys one unit at most (unit demand)
- There are two firms with constant marginal cost c, *whatever* the quality produced
- NB: So there is no *technological* reason for opting for low quality!

- One obtains the following result

- **Proposition** In the duopolistic quality and pricing game, one firm chooses maximal quality $\overline{s}$ and the other firm chooses minimal quality $\underline{s}$

  We derive the result using backwards induction (i.e. starting from the end and working backwards)

- **Step 1** *(Stage 2: picking a price given quality choices)*
  - Suppose w.l.o.g. that firm 1's quality s_1 is lower than that of firm 2, i.e. $s_1 < s_2$
  - This trivially implies that we must have $p_1 < p_2$, otherwise firm 1 would not sell, being more expensive and providing a good of lower quality
  - Assume also that prices are going to be set such that

$$p_1, p_2 < r$$

so that every consumer type (corresponding to any θ) is going to consume in equilibrium

- Given $s_1 < s_2$ and $p_1 < p_2$, there is a consumer type $\widehat{\theta}$ who is indifferent between buying products 1 and 2
- This type is defined by:

$$r + \widehat{\theta} s_1 - p_1 = r + \widehat{\theta} s_2 - p_2$$

Solving for $\widehat{\theta}$, we obtain:

$$\widehat{\theta} = \frac{p_2 - p_1}{s_2 - s_1} \text{ for } \widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$$

  - So the indifferent consumer is determined by the ratio of price differences to quality differences
  - Consumers of type $\theta \geq \widehat{\theta}$ buy the high quality product and remaining types buy the low quality product

- Assuming that prices are s.t.

$$\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right], \tag{1}$$

it follows that the profit functions of firms 1 & 2 are given by

$$\pi_1(p_1, p_2, s_1, s_2) = p_1 \left(\frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} \right)$$

and

$$\pi_2(p_1, p_2, s_1, s_2) = p_2 \left(\overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1} \right)$$

- Each firm's profit function is quasi-concave and each firm's optimal price can be found by taking FOCs w.r.t. the above defined profit functions

  We obtain:

$$p_1^* = \frac{1}{3}(\overline{\theta} - 2\underline{\theta})(s_2 - s_1)$$

  and

$$p_2^* = \frac{1}{3}(2\overline{\theta} - \underline{\theta})(s_2 - s_1)$$

- Note that equilibrium prices have several interesting features:
  - 1) Prices depend on quality differences
  - 2) Price of high quality firm increases in own quality but decreases in other's quality (more competition)
  - 3) Price of low quality firm increases in competitor's (high) quality (less competition) and decreases in own quality! (more competition)

- **Step 2***(Stage 1: picking a quality)*
- We can now substitute the obtained pricing strategies into the profit functions and solve for the optimal quality picked by each firm
  We have

$$\begin{aligned}
\widetilde{\pi}_1(s_1, s_2) &= \frac{1}{9}(\overline{\theta} - 2\underline{\theta})^2(s_2 - s_1) \\
\widetilde{\pi}_2(s_1, s_2) &= \frac{1}{9}(2\overline{\theta} - \underline{\theta})^2(s_2 - s_1)
\end{aligned}$$

- Note that $\widetilde{\pi}_1(s_1, s_2)$ monotonically decreases in s_1 while $\widetilde{\pi}_2(s_1, s_2)$ monotonically increases in s_2
- So the optimal qualities chosen are *maximally differentiated,* i.e.

$$\{s_1 = \underline{s}, s_2 = \overline{s}\}$$

Comments:

- There is maximum vertical differentiation
- Interestingly, one of the two firms decides to produce low quality even though *it is not more costly* to produce *high* quality
- The intuition is that setting maximally different qualities allows to maximally limit competition, which is advantageous to both firms

Comments (continued):

- But note that the firm producing the high quality makes higher profits, so that in a sequential quality setting game, the first firm would pick the maximal high quality, knowing that the second mover would pick the minimal quality
- Be aware that the extreme result obtained depends on our exact specification. The robust insight is however that vertical differentiation is used to advantageously limit competition