Price Discrimination

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Core Industrial Organisation Course for

Postgraduate Certificate in Competition and Regulatory Policy

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- What is price discrimination?
  - Price discrimination occurs when different consumers pay a different average price without this being justified by cost differences
    - Can be contentious; e.g. different drug prices in western countries
  - Examples
    - in-state/out-of-state tuition fees at public universities in USA
    - EU/non-EU tuition fees in the UK
    - passenger airline tickets
    - senior citizen, student discounts
    - doctors charging different prices to insured and the uninsured poor
- Price discrimination versus price differences
  - be aware that price differences and price discrimination are not the same
  - consumers at different distances from a plant pay different prices, but the price difference may be the result of difference in cost and not in prices of the good
  - alternatively, if price is same but cost is different, that too may be a form of price discrimination

Typology

- The general idea of price discrimination is that consumers differ in their willingness to pay and that as a consequence, a producer can often do better than a uniform price for all potential customers
- Consumer's willingness to pay (WTP) can be more or less easy to observe and the nature of this observability maps into a typology of price discrimination practices
  - First degree – WTP is perfectly observable
    - also called 'perfect' price discrimination – seller selects price for each individual buyer and consumers are left with no surplus
  - Second Degree – WTP is not observable but its distribution in known
    - includes 'non-linear pricing'; buyers self-select from a range of offers (menu) by seller
  - Third degree – WTP differs by groups, and group membership is observable
    - also known as group level discrimination; seller selects price for each identifiable group of buyers

First degree price discrimination

- In some cases, customers' willingness to pay is perfectly observable, in which case the firm can make personalized offers (*first degree price discrimination*)
- Examples
    - A tax accountant may observe perfectly the financial strength of her client and thus pick a price accordingly
    - Financial aid policies by universities (akin to price reduction), which are chosen after students filling in application packages providing extensive financial information
    - In the past, online platforms such as Amazon have experimented with individual pricing on the basis of past purchases on the website; the same book would not cost the same depending on who would log in (but they had to stop)

Third degree price discrimination

- In other cases, consumers belong to broad categories of consumers sharing certain basic *observable* characteristics
- In this case, the firm can make offers that are conditional on the observable characteristic of the customer (for example age, or frequency of consumption)
- This is known as *third degree price discrimination*
- Examples
  - Special customer cards for people above 60 or below 25, offered by for example transport companies or cinemas
  - Different prices for the same drugs in US and Canada
  - Subscription fees for Newspapers, for individuals or institutions

Second degree price discrimination

- Finally, sometimes there are no observable characteristics on the basis of which the firm might segment the market
- In this case, the firm has to rely entirely on self-selection in designing its pricing strategy
- This is known as *second degree price discrimination*
  - Two-part tarrif (e.g. membership fee plus usage fee)
  - Other non-linear pricing and quantity discounts
  - In general, the firm can propose a menu of offers in such a way that different customer types will self-select into different offers that are meant for their respective types (including quality discrimination where different price of alternative products do not fully reflect the quality differences)
  - Examples
    - Phone company contracts (Vodafone, etc)
    - Insurance contracts

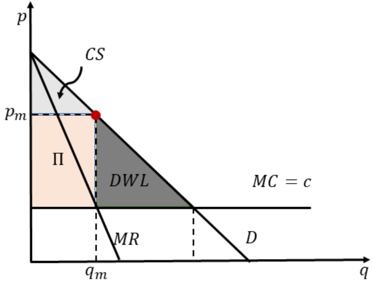
Other instruments of price discrimination

- Sometimes a given price discrimination practice does not unambiguously fall within one of these three categories
- Firms may use a variety of instruments or variables to discriminate among consumers
- Other instruments for price discrimination
  - Product tying – sell a product (or service) on the condition that the buyer buys another good from the seller, or not purchase the other good from a competitor
  - Discrimination through prices that differ over time (so-called intertemporal price discrimination)

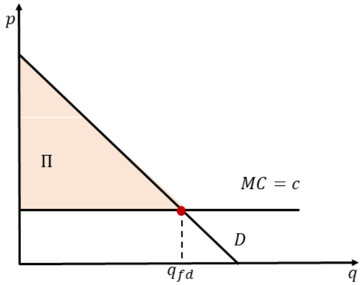
- Introduction (Typology)
  - Many different ways to price discriminate
  - First, second, third degree
  - (already done)
- Why and When – Incentives and necessary conditions
  - Why does a monopolist want to price discriminate (incentives)?
  - When can a monopolist be successful (necessary conditions)?
- First degree price discrimination
  - Consumer Surplus – Price equal to WTP
  - Consumer Surplus – Two-part tariff
- Third degree price discrimination
  - Two markets – uniform price vs two prices
  - Consumer welfare, profits, outputs
- Second degree price discrimination
  - Product versioning
  - Bundling

Single price monopolist

- Consider a monopolist setting a single (uniform) profit maximizing price; set price where marginal revenue is equal to marginal cost ($MR = MC$)
- Compared to the competitive level, output is lower and price is higher; consumer surplus, firm profit and efficiency loss (dead weight loss) are shown in the figure
- If the monopolist wants to sell to an additional buyer beyond q_m, as single price monopolist, she would need to lower her price to all the earlier payers – this has two effects on total revenue
  - her revenue increases by $p \cdot \delta q$ from selling an extra unit (let $\delta q = 1$), but it also decreases by $q \cdot \delta p$, where δp is the decrease in price to induce one more unit of sale
  - the second effect is larger and hence a single price monopolist would not move from the initial position (if it was not, then the monopolist was not maximizing her profit to start with)



- But what if the monopolist could charge lower price on just this last additional unit?
- Then she could increase the profit by $1 * p_l$ where p_l is the lower price charged on this extra unit without lowering the previous prices ($p_l = p_m - \delta p$)
- In fact, what if the monopolist could charge *a different price to each consumer* ... charge the maximum they are willing to pay (assuming the monopolist could figure out how much they are willing to pay)
  - compared to the uniform price monopolist, **profit would increase** to gobble up all the consumer surplus and what was previously dead weight loss (*perfect*!) – we will refer to this as **first degree price discrimination**
  - each consumer would pay according to the their willingness to pay (WTP), and total output, q_{fd} in this case would be equal to the output in a competitive case

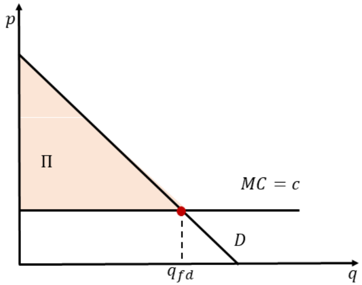


- Necessary conditions to discriminate
  - Market power – a firm must have some ability to set price above marginal cost (market power)
  - Differential willingness to pay/elasticities – there must be variation in WTP either across individuals or across units
  - Knowledge – A firm must be able to know or infer consumers willingness to pay
  - No arbitrage opportunities – a firm must be able to prevent arbitrage, i.e., prevent resale from one person (or group) to another

- Consumers buying at low price may not always be able to sell to those meant to purchase at a high price due to various conditions or actions by the monopolist
  - Service – services cannot be resold
  - Warranties – may be void upon resale
  - Contracts – resale may be forbidden
  - Single purpose product – customize so can't be reused for other purposes
    - make expensive alcohol for drinking vs cheaper for medicinal purposes and mix ingredients in latter so unfit for causal drinking
  - Transaction costs – transportation costs, tariffs
  - Vertical integration – seller may vertically integrate selectively with more price elastic downstream buyer (or industry) to prevent resale to less elastic one

- Consider the following case of vertical integration
  - Upstream monopolist sells raw material to two downstream firms (or industries), each facing different elasticity of demand
  - Say one downstream firm has more inelastic demand for the raw material compared to the other, and the monopolist wants to sell it for a higher price
  - If the other firm with more elastic demand can resell the raw material to the first firm, then the no arbitrage condition is missing
  - In this case, the upstream monopolist may vertically integrate with the firm with more elastic demand, sell it internally at the marginal cost, and prevent its new division from undercutting it
- Some evidence that Alcoa, that was the primary producer of aluminum ingot, may have had similar concerns and hence selectively vertically integrated into downstream industries depending on their elasticity of demand for aluminium
  - Perry, 1980, "Forward Integration by Alcoa: 1888-1930", *Journal of Industrial Economics* 29(1) 37-53

- Say each consumer has a unit demand and they differ in their WTP
- Monopolist faces downward sloping demand curve (graph shows WTP in descending order by consumers)
- Monopolist knows the WTP of each consumer and charges a price equal to WTP (case discussed earlier)
  - Often infeasible to know WTP of each consumer and to charge them a different price
  - A price discriminating monopolist produces at the perfectly competitive level
  - Highly profitable, no dead weight loss, minimizes consumer welfare (CS = 0) and maximizes total welfare (in form of profits)

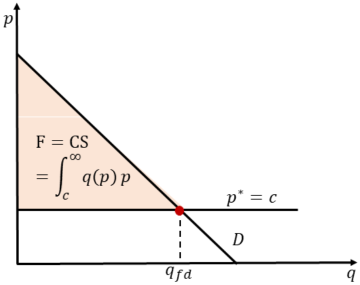


- Alternatively, say consumers are identical and have downward sloping demand curve
- Monopolist can still extract all the surplus (as in perfect discrimination) by using **two-part tariff** (technically, two-part tariff is the simplest form of second degree price discrimination)
- Charge a fixed fee (F) for the right to purchase (like an entry fee into a park) and then charge per unit of use (like fee for a ride) for total tariff of $T = F + V$
  - Then can extract all CS like the first degree price discrimination case if

$$F = \int_c^\infty q(p)d = CS$$

$$V = p^* q \qquad \text{where} \qquad p^* = c$$

  - (Note that if consumers are heterogenous then can still use same strategy and set $F_i = \int_c^\infty q_i(p)d = CS_i$)



Third Degree PD

Inverse elasticity rule

- Monopolist knows demand functions by groups, and there is group level heterogeneity in their price responsiveness
- Also referred to as multimarket price discrimination or market segmentation
- Cannot differentiate between consumers within a group (and cannot price discriminate within group)
- As long as arbitrage is not possible across groups, set different prices for each group based on the **inverse elasticity rule** – group with more elastic demand gets a lower price
- Examples
  - Different prices in different international markets (books, drugs, etc.)
  - Different prices for kids versus adults (restaurant menus), discounts for seniors
  - Location of a gasoline/petrol station (highway versus in town)

- Suppose there are two groups of consumers in two markets, and demand for each group is given by $q_1(p_1)$ and $q_2(p_2)$, and the marginal cost of production is c (common to both groups)
  - For example, $q_1(p_1)$ and $q_2(p_2)$ are demand functions in markets 1 and 2 and are given by

$$q_1(p) = 10 - p_1 \qquad \Rightarrow p_1 = 10 - q_1$$
$$q_2(p_2) = 10 - 2p_2 \qquad \Rightarrow p_2 = 10/2 - q_2/2$$

- If the monopolist can price discriminate, then she chooses two prices to solve the following problem

$$\max_{p_1, p_2} (p_1 - c)q_1(p_1) + (p_2 - c)q_2(p_2)$$

- Since demand in market i is independent of price in market j, then the monopolist can solve the joint problem as two separate problems of maximizing profits in each market i as $\max_{p_i}(p_i - c)q_i(p_i)$
- Thus, from the FOC $(\partial \pi_i / \partial p_i = 0)$ we get the condition

$$q_i^* + (p_i^* - c)\frac{dq_i^*}{dp_i} = 0$$

$$p_i^* + \frac{dq_i^*}{dp_i} q_i^* = c$$

where the expression on the left of the equality is MR and that on the right is mc and the superscript $*$ stands for optimal value under price discrimination

- If we now substitute the expression for elasticity, $\epsilon_i = \frac{dq_i}{dp_i}\frac{p_i}{q_i}$, we get

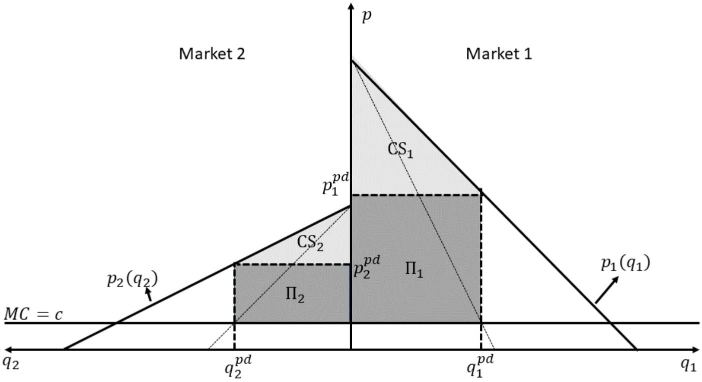
$$p_i^* \big(1 + \frac{1}{\epsilon_i}\big) = c$$

which is the inverse elasticity rule $((p_i - c/p_i) = -1/\epsilon_i)$, and so

$$\frac{p_1^*}{p_2^*} = \frac{1 + \frac{1}{\epsilon_2}}{1 + \frac{1}{\epsilon_1}}$$

- So the monopolist will set higher price in the market with less elastic demand — this is essentially the ‘Ramsey pricing rule’
  - Note, the condition holds only at optimal price and elasticity is generally a function of prices
  - $\epsilon_1 > \epsilon_2 \Rightarrow p_1^* > p_2^*$ (say $\epsilon_1 = -2, \epsilon_2 = -3 \Rightarrow p_1^* > p_2^*$)
  - If $\epsilon_1 \neq \epsilon_2$, monopolist is better off not setting the two prices to be equal
  - Your demand for food inside a stadium, in an airport, or other similar places would be relatively less elastic, and hence prices would be higher compared to when you can step out and have more choice

- A price discriminating monopolist in two markets
  - Market 1: demand curve is less steep (equivalently, inverse demand curve is steeper)
  - Price discriminating monopolist would set higher price in market 1
  - Numerical example[1] ...



[1] Change of notation: in what follows we list equilibrium price p_i^* as p_i^{pd} or as p_i^u for equilibrium price under price discrimination or under unform price, respectively.

Numerical example ...

- A price discriminating monopolist in two markets

$$q_1(p_1) = 10 - p_1 \qquad \Rightarrow p_1 = 10 - q_1$$
$$q_2(p_2) = 10 - 2p_2 \qquad \Rightarrow p_2 = 10/2 - q_2/2$$

  and let marginal cost be $c = 2$

- Monopolist's problem: choose p_1 and p_2 to maximize the total profit

$$\max_{p_1, p_2} (p_1 - c)q_1(p_1) + (p_2 - c)q_2(p_2)$$
$$\max_{p_1, p_2} (p_1 - c)(10 - p_1) + (p_2 - c)(10 - 2p_2)$$

- Since demand curves in each market are independent of price from the other market, can just treat it as two separate maximization problems facing the monopolist

$$\max_{p_1}(p_1 - c)(10 - p_1)$$
$$\max_{p_2}(p_2 - c)(10 - 2p_2)$$

- Solve separately in each market ($c = 2$)

Market 1

- $\max_{p_1} \Pi_1 = (p_1 - c)(10 - p_1)$
- FOC ($d\Pi_1/dp_1 = 0$) gives
  $10 - 2p_1 + c = 0$

$$p_1^{pd} = 6 \qquad q_1^{pd} = 4$$
$$\Pi_1^{pd} = 16 \qquad CS_1^{pd} = 8$$
$$(CS/q)_1^{pd} = 2$$

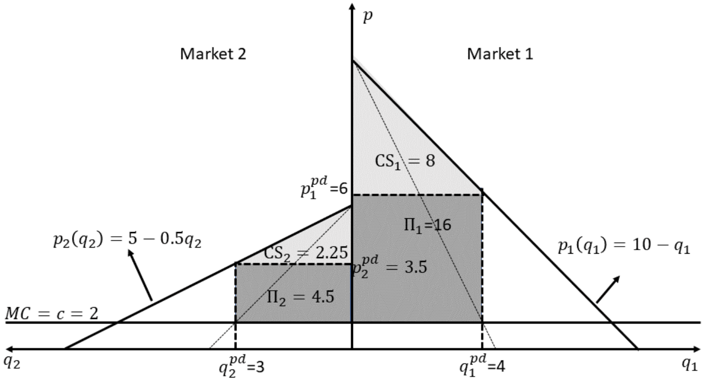
Market 2

- $\max_{p_2} \Pi_2 = (p_2 - c)(10 - 2p_2)$
- FOC ($d\Pi_2/dp_2 = 0$) gives
  $10 - 4p_2 + 2c = 0$

$$p_2^{pd} = 3.5 \qquad q_2^{pd} = 3$$
$$\Pi_2^{pd} = 4.5 \qquad CS_2^{pd} = 2.25$$
$$(CS/q)_2^{pd} = 0.75$$

- Total output, profit and weighted average of CS ($\frac{CS_1 + CS_2}{q_1 + q_2}$)
  - $q^{pd} = 4 + 3 = 7$
  - $\Pi^{pd} = 16 + 4.5 = 20.5$
  - $(CS/q)^{pd} = (8 + 2.25)/(4 + 3) = 1.46429$

- A price discriminating monopolist in two markets
  - Market 1: demand curve is less steep (equivalently, inverse demand curve is steeper)
  - Price discriminating monopolist would set higher price in market 1
  - Numerical example ...



- How does this compare to when there is no price discrimination?
- If the monopolist sets a uniform price p^u then

$$\max_{p_1,p_2}(p_1 - c)(10 - p_1) + (p_2 - c)(10 - 2p_2) \qquad \text{s.t. } p_1 = p_2 = p$$

or, equivalently

$$\max_{p}(p - c)(20 - 3p)$$

Market 1		Market 2	
$p_1^u = 4.33$	$q_1^u = 17/3 = 5.667$	$p_2^u = 4.33$	$q_2^u = 4/3 = 1.333$
$\Pi_1^u = 13.222$	$CS_1^u = 16.0556$	$\Pi_2^u = 3.111$	$CS_2^u = .444$
$(CS/q)_1^u = 2.833$		$(CS/q)_2^u = 0.333$	

- Total output, profit and weighted average of CS ($\frac{CS_1+CS_2}{q_1+q_2}$)
  - $q^u = 5.667 + 1.333 = 7$
  - $\Pi^u = 13.22 + 3.11 = 16.33$
  - $(CS/q)^u = 16.5/7 = 2.357$

- Profits
  - Increases under price discrimination compared to under uniform pricing ($\Pi^{pd} = 16 + 4.5 = 20.5 > \Pi^{u} = 13.222 + 3.111 = 16.333$)
  - This is always true ($\Pi^{pd} \geq \Pi^{u}$) ... else can always set prices under pd to be the same and earn at least as much as under uniform pricing
  - Total output is the same in this example, but this is not a general result; if total output is higher under price discrimination, total welfare (CS + profit) may increase
- Distributional effects
  - Under price discrimination, price is higher and quantity is lower in market 1 compared to uniform pricing ($p_1^{pd} = 6, q_1^{pd} = 4$ and $p_1^{u} = 4.33, q_1^{u} = 5.667$)
  - The opposite is true in market 2, price is lower and output is higher under price discrimination ($p_2^{pd} = 3.5, q_2^{pd} = 3$ and $p_2^{u} = 4.33, q_2^{u} = 1.337$)
- Consumer surplus
  - Per unit consumer welfare is lower for group 1 under PD compared to uniform price (2 vs 2.833 respectively), while for group 2 it is higher under PD (.75 vs .33); group 1 loses and group 2 gains under PD
  - Weighted average of CS under PD is 1.464 and under uniform pricing is 2.357; overall avg CS is lower but this is specific to this example

- Price discrimination always increases firm profit
- Consumers in the inelastic market would weakly prefer uniform pricing over price discrimination; those in the more elastic market would prefer price discrimination
- Under both systems, price is above the marginal cost and there is some output inefficiency
  - There is one DWL area under uniform pricing from the aggregate market, and two such areas in separate markets with their own monopoly prices; in general can't say if the sum of the latter two is larger or smaller
  - If price discrimination increases total output, then it may increase total welfare (if total output declines, then welfare will decrease)
- Banning price discrimination
  - If uniform price policy leads to a price such that only the inelastic group is served, then allowing price discrimination improves welfare
  - The inelastic group would be indifferent and the consumer surplus of the elastic group would increase, as would firm profit

- Second degree discrimination arises when the monopolist knows there there is consumer heterogeneity in WTP, but cannot tell apart different consumers
- In this case the monopolist can charge non-liner prices or use menu of bundles for consumers to choose from
  - Example of a non-linear price was in the two-part tariff we discussed earlier
  - Alternatively, the monopolist can charge different prices based on characteristics of the bundle; example – quantity discount (buy one get second free, or different unit price for different sized packages) or **product versioning** (higher/newer quality vs lower version)
  - Bundle products together i.e. use bundling and tie-ins: meal deals (drink, sandwich and a cookie), entertainment and news packages on cable TV offering, multiple software in a suite (e.g. MS Office)

- Positive quality enhancements to separate high valuation customers from those with low valuation (i.e. WTP)
  - More leg room, better menu, more comfort, priority check-in on business flights
  - Hardback books with early release for libraries and literature buffs
  - Software with enabled features in the ‘professional’ version
  - Box and house seats and other amenities for theater fans
- Sell a higher quality product to those with higher WTP at a higher price, and those with lower WTP a product with lower quality and lower price
- Use negative quality detriments to discourage high valuation customers from selecting cheaper version
  - Cramped economy class seats
  - Late release in paper back edition
  - Disabled features in home edition
  - Stalls and balconies
- How would a monopolist set their prices?

SECOND DEGREE PD
PRODUCT VERSIONING

DAMAGED GOODS

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Manufacturers may intentionally damage a portion of their goods in o
price discriminate. Many instances of this phenomenon are observed.
result in a Pareto improvement.

2.2 IBM LASERPRINTER E

In May 1990, IBM announced the introduction of the LaserPrinter E, a lower cost alternative to its popular LaserPrinter. The LaserPrinter E was virtually identical to the original LaserPrinter, except that the E model printed text at 5 pages per minute (ppm), as opposed to 10 ppm for the LaserPrinter. According to Jones (1990), the LaserPrinter E uses the same "engine" and virtually identical parts, with one exception:

> The controllers in our evaluation unit differed only by virtue of four socketed firmware chips and one surface mounted chip. PC Labs' testing of numerous evaluation units indicated that the LaserPrinter E firmware in effect inserts wait states to slow print speed. . . . IBM has gone to some expense to slow the LaserPrinter in firmware so that it can market it at a lower price.

- In third degree price discrimination, monopolist knows who belongs to which group and sets price for the group accordingly
- By contrast, when the monopolist knows that there are multiple types of consumers – ie they differ in their WTP — but does not know who belongs to which group, then set up price schemes so that each consumer selects into the right group
- The trick is to design them in such a way so that (i) consumers self-select into the right package/bundle designed for them and, (ii) they don't walk away from the offers
  - **Participation constraint**
  - **Incentive compatible constraint**
- The selection issue/constraints above apply to bundles as well as two-part tariffs when consumer belong to different groups; the selection of a bundle gives a signal about the consumer type
- Example ...

- Say there are two types of customers: those that have high WTP and those with low WTP — call them h and l type (business travellers versus vacationers)
- Ideally, the firm would like to set different price for each customer and charge them their maximum WTP, but cannot tell them apart – the high type could lie and pay price meant for the low type
- Accordingly, the monopolist makes two versions of their product – call them H and L (business versus economy seating) – for the two types of customers and sets the price per seat to be p_H and p_L
- Set the prices p_H and p_L such that each customer purchases an item (**participation constraint**), and in fact purchases the version meant for them (**incentive compatible constraint**)

$$v_l(L) - p_L \geq 0 \qquad (1 – \text{PC1})$$

$$v_h(H) - p_H \geq 0 \qquad (2 – \text{PC2})$$

$$v_l(L) - p_L \geq v_l(H) - p_H \qquad (3 – \text{IC1})$$

$$v_h(H) - p_H \geq v_h(L) - p_L \qquad (4 – \text{IC2})$$

where $v_i(\cdot)$ is the utility type i gets from consuming a unit of H or L

- Constraints are

$$
\begin{aligned}
v_l(L) - p_L &\geq 0 && (1 – \text{PC1})\\
v_h(H) - p_H &\geq 0 && (2 – \text{PC2})\\
v_l(L) - p_L &\geq v_l(H) - p_H && (3 – \text{IC1})\\
v_h(H) - p_H &\geq v_h(L) - p_L && (4 – \text{IC2})
\end{aligned}
$$

  where $v_i(\cdot)$ is the utility type i gets from consuming a unit of H or L
- Constraint (1) and (2)
  - Ensure that each type of consumer is better off purchasing the product intended for them at the the given price, than not participating in the transaction
  - So l type gets more utility from purchasing a unit of L at price p_L than doing without it; similarly for h type for the other product
- Constraint (3) and (4)
  - Ensure that each type of consumer is better off selecting the product for them at that price than the the other product at different price
  - So h type should get more utility from purchasing a unit of H net of its price $(u_h(H) - p_H)$ than she would get from purchasing lowered price L product $(u_h(L) - p_L)$; similar for l type

- Numerical example – say the consumer utilities are as follows

$$v_h(H) = 200 \qquad v_h(L) = 80$$
$$v_l(H) = 100 \qquad v_l(L) = 60$$

- Then the two set of constrains imply that prices should be such that

$$60 - p_L \geq 0 \qquad (1 – \text{PC1})$$
$$200 - p_H \geq 0 \qquad (2 – \text{PC2})$$
$$60 - p_L \geq 100 - p_H \qquad (3 – \text{IC1})$$
$$200 - p_H \geq 80 - p_L \qquad (4 – \text{IC2})$$

- Solution – pick the highest prices that satisfy the constraints
  - To satisfy the first participation constraint, $p_L \leq 60$. So to maximize the profit, set it at the highest allowed price of $p_L = 60$, as setting it any lower only decreases the total profit
  - Next p_H cannot be more than 200 else the h type will not buy H (constraint 2)
  - Further, since $p_L = 60$, then constraint 4 is satisfied if p_H is less than or equal to 180 (so the previous constraint is redundant)
  - Finally since $p_L = 60$, then p_H should be more than 100, otherwise l type can buy H (constraint 3)
  - Putting the last two constraints together, p_H should be more than or equal to 100 and less than or equal to 180. To maximize profits, set it to the highest allowed, so $p_H = 180$
- Solution – $p_L = 60, p_H = 180$

- Some general observations
  - At the profit maximizing prices, constrains (1 - PC1) and (4 - IC2) were **binding**
  - These were the participation constraint of the low value consumer and the incentive compatibility constraint of the high value consumer
  - Thus the low value consumer is made indifferent between purchase or not (PC1), and the high value consumer is made indifferent between product versions (IC2)
  - The incentive computability constraint of high value consumer made the participation constraint of the high value consumer **reductant** (and not binding)
  - By setting $p_L = v_l(L)$ and binding PC1 constraint, the low value consumer has zero consumer surplus
  - Similarly, since PC2 is not binding, the high value consumer derives a positive consumer surplus
- These conditions generally hold in such problems, even if there were more versions

- Another type of product offers – **Bundling** which allows for indirect price discrimination
- When two or more products are combined as a package
  - Excel spreadsheet with a word processor
  - News channel with movies channel
  - Guitar lessons with drumming lessons
- Mixed and Pure Bundling
  - Mixed – individual components or as a bundle with discounted price
  - Pure – only offer bundles and cannot buy separately
- Correlated preferences
  - Negatively correlated – Consumer A is willing to pay more for guitar lessons and less what they would pay for drumming lessons, but consumer B is willing to pay more for drumming lessons compared to guitar lessons
  - Positively correlated – Two room mates such that if one is willing to pay more than the other for the movies channel, it is the same roommate who is also willing to pay more for the news channel relative to the other roommate
  - If preferences are **negatively correlated** then bundling can work

- Say two products News and Movies and two consumers A and B
- Say WTP as follows

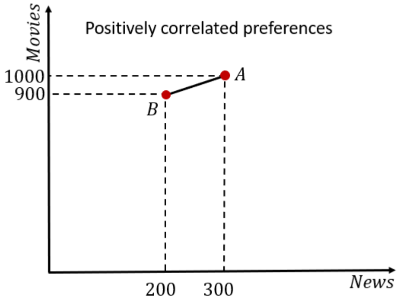
+ve corr	WTP Movies	WTP News
Consumer A	1000	300
Consumer B	900	200

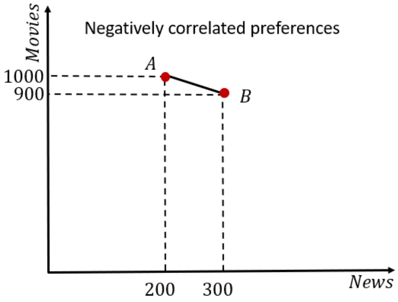
  - While WTP for movies is generally higher than for news, the key that consumer A is willing to pay more for movies and news while consumer B is willing to pay less for both; preferences are **positively correlated**

- Say WTP as follows

-ve corr	WTP Movies	WTP News
Consumer A	1000	200
Consumer B	900	300

  - While WTP is again generally higher for movies than for news, the key is that while for movies A is willing to pay more than B, for news the opposite is true that A is willing to pay less than B for the news; preferences are **negatively correlated**


Movies
Positively correlated preferences
1000
900
A
B
200
300
News


Movies
Negatively correlated preferences
1000
900
A
B
200
300
News

With and without bundling, positive correlation

- Say the preference are positively correlated, how would the prices be set if
  - (1) monopolist can set a different price for each consumer (and for each product)
  - (2) is forbidden by law to discriminate among consumers and cannot bundle
  - (3) is forbidden by law to discriminate but can bundle

+ve corr	WTP Movies	WTP News
Consumer A	1000	300
Consumer B	900	200

  - (1) Trivially prices would be 1000, 900, 300 and 200 for a total profit of 2400
  - (2) Without bundling, monopolist will charge $\min(1000, 900)$ for movies package and $\min(300, 200)$ for the news package for a total profit of 2200
  - (3) With bundling, monopolist will charge $\min(1000 + 300, 900 + 200) = 1100$ for a total profit of 2200
- Note that profit did not increase by bundling

With and without bundling, negative correlation

- Say the preference are negatively correlated, how would the prices be set if
  - (1) monopolist can set a different price for each consumer (and for each product)
  - (2) is forbidden by law to discriminate among consumers and cannot bundle
  - (3) is forbidden by law to discriminate but can bundle

-ve corr	WTP Movies	WTP News
Consumer A	1000	200
Consumer B	900	300

  - (1) Trivially prices would be 1000, 900, 300 and 200 for a total profit of 2400
  - (2) Without bundling, monopolist will charge $\min(1000, 900)$ for movies package and $\min(200, 300)$ for the news package for a total profit of 2200
  - (3) With bundling, monopolist will charge $\min(1000 + 200, 900 + 300) = 1200$ for a total profit of 2400
- Note that profit increased with bundling, in fact it restored to the level of price discrimination
- This is an example of **indirect price discrimination**

- As for all types of PD, some consumers may be harmed, but others benefit if they would not otherwise be supplied
  - Does output rise?
- Other reasons for bundling include
  - Technical efficiency
  - Entry deterrence (against specialist entrant)
  - Foreclosing demand for products of existing rivals
- Bundling is more of a concern in oligopoly (or monopoly threatened by entry) than in pure (unchallenged) monopoly

- Consumers have different willingness to pay and monopolist attempts to capture the consumer surplus
- First degree – monopolist's profits are higher and consumers surplus zero
- Third degree – monopolist's profits are higher and consumers may be better off
- Second degree – monopolist's cannot distinguish between types and uses other methods to price discriminate

Extra Slides on Bundling

- We will compare profits and consumer surplus in
  - Separate selling
  - Pure bundling
  - Mixed bundling
- We will focus on when
  - Products are unrelated (as opposed to complements or substitutes)
  - A given consumer's valuation for products are uncorrelated (as opposed to correlated across products)

- As a reminder how bundling works, consider the following example
  - Say each product is produced at zero cost
  - Consumer demands one unit of each product
  - Two consumers with negatively correlated valuations as given below

	Product 1	Product 2
Consumer A	3	2
Consumer B	2	3

- Separate selling:
  - The firm sells each product at price 2,
  - Sells both goods to both consumers
  - Obtains a profit of 8
- Pure bundling:
  - The firm sets a price of 5 for the bundle.
  - Both consumers buy the bundle,
  - Yielding profits of 10
- So profits can increase with bundling if consumers valuations are negatively correlated
  - Is bundling also profit increasing in a more general model?

Lets consider a more general model

- Monopolist producing two goods A and B at zero cost
- Each consumer is identified by a vector (θ_A, θ_B)
  - θ_i is a consumer's valuation of good i
  - A consumers valuations have a joint uniform probability density $f(\theta_A, \theta_B)$ with support on a unit square $[0, 1] \times [0, 1]$ and where valuations for A and B are independent
  - Mass of consumers is normalized to 1
- A consumer's valuation of the bundle AB is $\theta_{AB} = \theta_A + \theta_B$
  - This is realistic for unrelated goods, not for complements or substitutes
  - for complements $\theta_{AB} > \theta_A + \theta_B$
  - for substitutes $\theta_{AB} < \theta_A + \theta_B$

- **Proposition.** If consumers have heterogeneous but uncorrelated valuations for two products, then
  - *Profits are higher under pure bundling compared to separate selling*
  - *The monopolist inflates demand by selling the bundle cheaper than the combined price under separate selling*

- **Step 1:** Consider separate selling
    - Each of the goods is sold separately and priced independently
    - Clearly, the profit maximizing price of product i solves

$$\max p_i^s (1 - p_i^s)$$

which yields optimal prices $p_A^s = p_B^s = \frac{1}{2}$

- So the monopoly's profits are given by

$$\pi^s = \frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2} = \frac{1}{2}$$

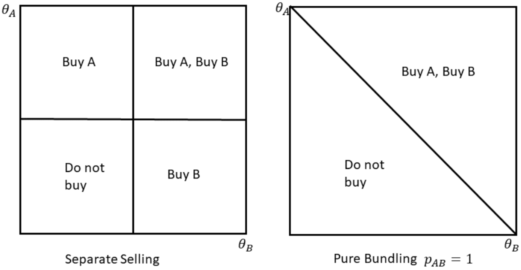
- And CS under separate selling is

$$\begin{aligned} CS^s &= \int_{p^s}^{1} (\theta_A - p) d\theta_A + \int_{p^s}^{1} (\theta_B - p) d\theta_B \\ &= (p^s - 1)^2 = \frac{1}{4} \end{aligned}$$

- **Step 2:** Consider now pure bundling
  - Denote the price of the bundle by p_{AB}
  - The monopolist can replicate the previous solution by setting

$$p_{AB} = p_A^s + p_B^s$$

    where p_K^s is opitimal price under separate selling
  - The above price yields the same profits as under separate selling, but the identity of the buying consumers changes (now $\theta_A + \theta_B \geq p_{AB}$)



- Let us now characterize the optimal price under pure bundling
- Under pure bundling, for a given price p_{AB}, the consumers who do not buy are those with $\theta_A + \theta_B < p_{AB}$
  - The mass of such consumers is given by $\frac{1}{2}\left(p_{AB}\right)^2$
  - To see this:
    - note that the lower-right vertex of the triangle has $\theta_A = 0$ and $\theta_B = p_{AB}$
    - note that the upper left vertex of the triangle has $\theta_B = 0$ and $\theta_A = p_{AB}$
- Hence the demand at price p_{AB} is given by $1 - \frac{1}{2}\left(p_{AB}\right)^2$
- The profit maximizing price solves

$$\max p_{AB}\left(1 - \frac{1}{2}\left(p_{AB}\right)^2\right)$$

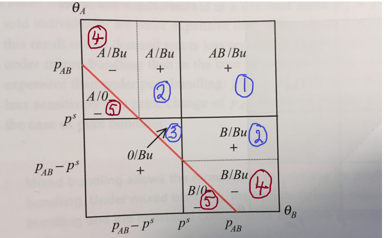
- Taking FOCs yields

$$p^b_{AB} = \sqrt{\frac{2}{3}} \simeq .82 < 1 = p^s_A + p^s_B$$

  - The monopolist sets p^b_{AB} for the bundle which is lower than $p^s_A + p^s_B$
  - For those who buys both goods under both policies, their surplus increases
  - The profit is higher than under separate selling

$$\begin{aligned} \pi^b &= p_{AB}\left(1 - \frac{1}{2}(p_{AB})^2\right) \\ &= \sqrt{\frac{2}{3}}\left(1 - \frac{1}{2}\frac{2}{3}\right) \simeq .544 > \frac{1}{2} \end{aligned}$$

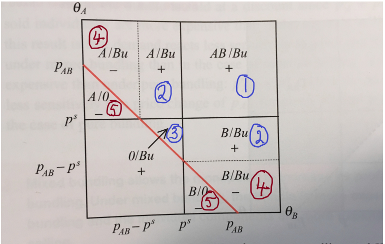
- But how is consumer welfare affected when going from separate selling to pure bundling?
- Some consumers benefit from the transition, while others lose
  - ('+') Those consumers who have a high valuation for both goods prefer pure bundling – their welfare increases under bundling (marked with '+' in the picture)
  - ('−') Those consumers who have very asymmetric valuations prefer separate selling – their welfare decreases under bundling (marked with '−' in the picture)



'X/Y' means that the consumer buys X under separate selling and Y under pure bundling; '+' ('−')means that pure bundling increases (decreases) consumer surplus with respect to separate selling.

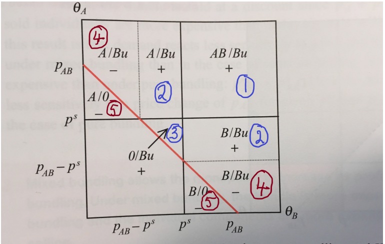
SECOND DEGREE PD
MODEL - PURE BUNDLING VS SEPARATE SELLING

- Let us here consider consumers who prefer *pure bundling*
- There are three subcategories (see '+' in the picture)
  - (1) Consumers with $\theta_A, \theta_B \geq p^s$ buy both goods under both regimes but buy at a discount under pure bundling
  - (2) Consumers with $\theta_i \geq p^s, \theta_j < p^s$ and $\theta_j \geq p^b_{AB} - p^s$ buy one more good under pure bundling and increase their surplus to $\theta_A + \theta_B - p^b_{AB} \geq \theta_i - p^s$ (this inequality being equivalent to $\theta_j \geq p^b_{AB} - p^s$)
  - (3) Consumers with $\theta_A + \theta_B \geq p^b_{AB}$ and $\theta_A, \theta_B < p^s$ buy nothing when goods are sold separately but buy the bundle under pure bundling



'X/Y' means that the consumer buys X under separate selling and Y under pure bundling; '+' ('−')means that pure bundling increases (decreases) consumer surplus with respect to separate selling.

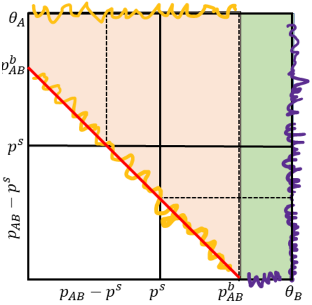
- Let us now consider consumers who prefer *separate selling*
- These can be divided into two subgroups (see '−' in the picture)
  - (4) Consumers with $\theta_i \geq p^s$, $\theta_j < p^s$, $\theta_A + \theta_B \geq p^b_{AB}$ and $\theta_j < p^b_{AB} - p^s$ buy one more good under bundling, but their consumer surplus is reduced from $\theta_i - p^s$ to $\theta_A + \theta_B - p^b_{AB}$
  - (5) Consumers with $\theta_i \geq p^s$, $\theta_j < p^s$ and $\theta_A + \theta_B < p^b_{AB}$ buy one good under separate selling and do not buy under bundling



'X/Y' means that the consumer buys X under separate selling and Y under pure bundling; '+' ('−')means that pure bundling increases (decreases) consumer surplus with respect to separate selling.

- Under pure bundling, $CS^b =$

$$\int_0^{p^b_{AB}} \left(\int_{p^b_{AB}-\theta_B}^{1} \left(\theta_A + \theta_B - p^b_{AB}\right) d\theta_A \right) d\theta_B$$
$$+ \int_{p^b_{AB}}^{1} \left(\int_0^1 \left(\theta_A + \theta_B - p^b_{AB}\right) d\theta_A \right) d\theta_B$$



- To understand the above expression let $u = \theta_A + \theta_B - p^b_{AB}$
- Integrate u over the green region where θ_A varies from 0 to 1 and θ_B varies from p^b_{AB} to 1 (second integral line)
- Integrate u over the light brown area; θ_B ranges from 0 to p^u_{AB}; θ_A must stay above the optimal price line (line p^b_{AB}) and hence θ_A is from $p^b_{AB} - \theta_B$ to 1 (first integral line)

- Which gives

$$
\begin{aligned}
CS^b = &= 1 - p^b_{AB} + \frac{1}{6}\left(p^b_{AB}\right)^3 \\
&= 1 - \frac{8}{27}\sqrt{6} \simeq .27 \\
&> \frac{1}{4} = CS^s
\end{aligned}
$$

- So the aggregate consumer surplus is larger under pure bundling compared to under separate selling
- The main intuition: pure bundling increases the number of consumers who are served (keep in mind some consumers were worse off)

- Lets now compare to mixed bundling
  - Products can bought separately *or* as a bundle
  - This gives consumers more freedom
  - The original option (single product purchase) is now back on the table
  - And the bundle option presumably still comes with a discount
  - One would think that this is going to be:
    - for sure more attractive than pure bundling to consumers
    - probably less attractive for the monopolist
    - Wrong! It's more or less the other way round
    - Intuition: self-selection incentives
- **Proposition:** Under mixed bundling
  - The bundle is more expensive than under pure bundling, and the goods when bought individually, are more expensive than under separate bundling
  - Mixed bundling allows the monopolist to increase its profits even further than pure bundling

- **Step 1:** Identify consumers that are indifferent between choices
  - Consumers who are indifferent between buying product $k = A, B$ and not buying anything satisfy $\widehat{\theta}_k = p_k$
  - Consumers who are indifferent between buying product k and the bundle AB satisfy

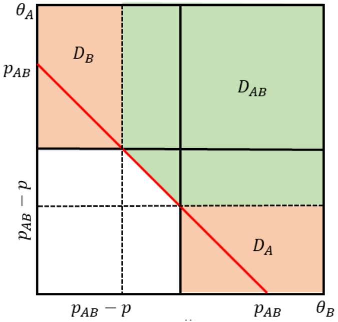
$$\theta_k - p_k = \theta_A + \theta_B - p_{AB}$$

  - We restrict ourselves to (symmetric) strategies under which $p_A = p_B = p$
- **Step 2:** Identify demand for A,B and bundle AB

- As long as the bundle is offered at a discount (i.e. $p_{AB} < 2p$), then the demands for products A and B and the bundle AB are

$$D_A(p, p_{AB}) = D_B(p, p_{AB}) = (1-p)(p_{AB}-p)$$

$$D_{AB}(p, p_{AB}) = (1 - p_{AB} + p)^2 - \frac{1}{2}(2p - p_{AB})^2$$



- **Step 3:** Write out the profit function and compute optimal prices p_{AB} and p
  - If prices are s.t the demands for individual products and the bundle are positive, then

$$\begin{aligned}\pi\left(p_A, p_B, p_{AB}\right) &= pD_A(p, p_{AB}) + pD_B(p, p_{AB}) \\ &\quad + p_{AB}D_{AB}(p, p_{AB}) \\ &= 2p(1-p)\left(p_{AB}-p\right) \\ &\quad + p_{AB}\left[\left(1-p_{AB}+p\right)^2 - \frac{1}{2}\left(2p-p_{AB}\right)^2\right]\end{aligned}$$

  - Find price via the usual FOC for each of the two prices

$$\frac{\partial \pi}{\partial p} = 2(2-3p)(p_{AB}-p) = 0$$

  - It follows that

$$p_A^m = p_B^m = \frac{2}{3}$$

- Taking partial wrt p_{AB} and evaluating at $p = \frac{2}{3}$, we obtain

$$\left.\frac{\partial \pi}{\partial p_{AB}}\right|_{p=\frac{2}{3}} = \frac{7}{3} - 4p_{AB} + \frac{3}{2}\left(p_{AB}\right)^2 = 0$$

- The only admissible solution (satisfying $p_{AB} < 2p$) is given by

$$p_{AB}^{m} = \frac{1}{3}(4 - \sqrt{2})$$

- It follows that the monopolist's profit under mixed bundling is given by

$$\begin{aligned} \pi^m &= \pi\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}(4 - \sqrt{2})\right) \\ &\simeq .549 \end{aligned}$$

- Note that this profit under mixed bundling is greater than that under pure bundling (.544) which in turn in greater than that under separate selling (.500)

- How about the welfare of consumers under mixed bundling?
- We obtain the following result

- **Proposition:** Consumers may be worse off under mixed bundling than under pure bundling

  - We do not prove the result
  - Recall we wrote: "*Under mixed bundling, the bundle is more expensive than under pure bundling and the goods, when bought individually, are more expensive than under separate bundling*"
  - In our current setup one can furthermore show that CS under mixed bundling is however larger than under separate selling ($CS^b < CS^m < CS^s$)

- Overall, results in this section depend on products being independent, valuations being from uniform distribution and not correlated, and marginal costs being zero

- It can be shown that the attractiveness of pure bundling to the firm decreases as the correlation between valuations becomes larger (going from negative to positive)
  - The intuition is that bundling serves to induce extra consumption than truly desired by consumers
  - By selling a bundle at a low price, the monopolist changes the behavior of consumers who have a very low value for one of the two goods
- Let us resolve part of the model (separate and pure bundling) under (perfect) positive correlation
- Correlated valuations
  - Assume $\theta_B = \rho\theta_A + (1 - \rho)(1 - \theta_A)$, with $\rho \in [0, 1]$
  - If $\rho = 1$, there is perfect positive correlation (all the consumers lie on the 45^o line)
  - if $\rho = 0$, they are negatively correlated (all consumers lie on the other diagonal given by $\theta_A + \theta_B = 1$)
  - Let $\rho = 1$; We will show that pure bundling and separate selling yield the same profit for the firm

- Consider first separate selling
  - Here, the firm maximizes $(1 - p_i)p_i$ for each good and sets $p_i = \frac{1}{2}$
  - This yields aggregate profits of $\pi^s = \frac{1}{2}$
- Consider now pure bundling
  - A consumer's valuation for the bundle is $2\theta_A$
  - Consumers who do not buy are those with $2\theta_A \leq p_{AB}$
  - There are $1 - \frac{1}{2}p_{AB}$ of these
  - To see this, note that all consumers can be represented on a line with $\theta \in [0, 1]$ (where $\theta = \theta_A = \theta_B$)
  - Consumers who buy the bundle are those with $2\theta \geq p_{AB}$, i.e. $\theta \geq \frac{1}{2}p_{AB}$

- So the optimal price for the (pure) bundle solves

$$\max p_{AB}(1 - \frac{1}{2}p_{AB})$$

- The optimal price $p^b_{AB} = 1$
- Thus all consumers with $\theta \geq \frac{1}{2}$ buy the bundle, yielding a profit $\pi^b(1) = \frac{1}{2}$
- So the firm's profits are the same under separate selling and pure bundling
    - Intuition: Under perfect correlation, pure bundling does not/cannot inflate demand
    - The consumers who buy the bundle are exactly the same as the consumers who would buy the two goods under separate selling