MONOPOLY AND OLIGOPOLY

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Core Industrial Organisation Course for

Postgraduate Certificate in Competition and Regulatory Policy

- Monopoly
  - What is a monopoly
  - Monopoly pricing
  - Welfare loss
  - Multiproduct monopoly
- Oligopoly with homogenous products
  - Cournot (quantity) competition
  - Stackelberg (leader/follower) competition
  - Bertrand (price) competition

What is a monpoly?
Market Definition

- What is a monopoly?
  - It is a firm that is “alone” in serving a given market
  - A market is defined by a product or a collection of products that are at most slightly differentiated (no need to impose perfect substitution)
  - The definition of a market should neither be too broad nor too narrow
  - A market can be defined by all its three dimensions – product, geographical area, and time

- One definition used by competition authorities to figure out a market is based on the *hypothetical monopoly test* – called the SSNIP test
  - *A market is the smallest product (and/or service) group such that a hypothetical monopolist (or cartel) controlling that product group could profitably sustain a Small and Significant Non-transitory Increase in Prices (SSNIP)*
  - It was first introduced by the US Department of Justice Merger Guidelines in 1982
  - Effectively, one checks whether the hypothetical monopolist could increase the price by 5-10% above competitive levels
  - If there exists a product substitute outside this group to which most consumers could switch and thereby make the price increase unprofitable, then the market definition should be increased so as to include this alternative product
  - Repeat the process until an SSNIP is sustainable
  - The SSNIP test often involves interviewing consumers

- Suppose a monopolist is selling three products A,B, C with marginal costs of 5,4,and 4 – the initial situation (prices and quantity) is depicted in the middle section of the table
- Say there is a 10% increase in the price of A (but not other products), and quantity demanded is as shown in the right panel of the table? Is this profitable?

		Before 10% price↑			After 10% price↑		
Product	MC	P	Q	Π	P	Q	Π
A	5	10	1000		11	800	
B	4	13	800	17700	13	900	18900
C	4	9	1100		9	1200	

  - Old profit 17,700 and new profit 18,900
  - In the example the three products are part of a relevant market
  - Had the price increase not been profitable one would have had to add other substitutable products until the price increase becomes profitable

- In the previous example we did not indicate how the prices were determined
- We next
  - look at demand curve and consumer welfare
  - see how firms set prices
  - describe and compare with perfect competition

- Consumer demand is a fundamental concept of a market
- It originates in rational optimization by consumers
  - A much used demand function is the *linear demand function*

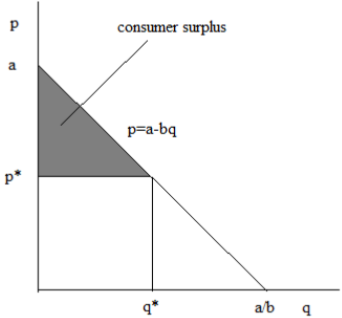
$$q(p) = \frac{a}{b} - \frac{p}{b}$$

which is more often written as the *inverse (linear) demand function:*

$$p(q) = a - bq$$

- Another example would be $q(p) = ap^{-\epsilon}$, where $\epsilon > 1$ and $a > 0$
  - This displays constant price elasticity of $-\epsilon$
  - The corresponding inverse demand is $p(q) = q^{-\frac{1}{\epsilon}} a^{\frac{1}{\epsilon}}$

- How well off are consumers or firms given outcomes? Compute equilibrium prices and quantities (p^*, q^*) in a specific market
- Consumer welfare is defined as *consumer surplus*, which is the net benefit that the consumer derives from being able to consume a good or purchase a service
  - It sums up the difference across units consumed between
    - what the consumer would be willing to pay
    - and what the consumer actually pays (the price)
  - The figure shows consumer surplus given price p^* and corresponding linear demand $q^* = \frac{a}{b} - \frac{p^*}{b}$
  - Consumer surplus is the area under the inverse demand curve and above the market price



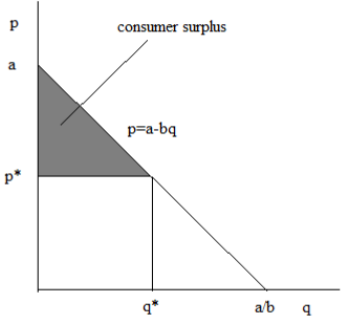
- Given that the demand curve is linear in this example, use knowledge of geometry to compute CS (area of a triangle is $(1/2) \times Base \times Height$)
  - Area of triangle

$$CS(p^*, q^*) = \frac{(a - p^*)q^*}{2}$$

    and substitute $p^* = a - bq^*$ to obtain

$$CS(p^*, q^*) = \frac{b\,(q^*)^2}{2}$$

  - More generally, need to integrate to obtain area under the demand curve
  - Consumer surplus is the area under the inverse demand curve and above the market price



- More generally, need to integrate to obtain area under the demand curve
  - Step 1. Identify the quantity consumed given price p^*. Given our linear demand, we have

$$q^*(p^*) = \frac{a}{b} - \frac{p^*}{b}$$

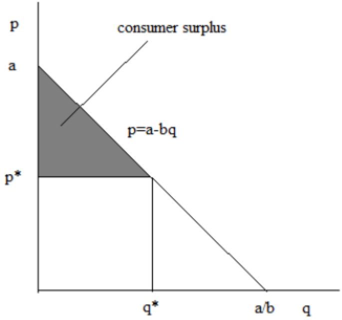
  - Step 2. Calculate

$$CS(p^*, q^*) = \int_0^{q^*} (a - bq)dq - p^*q^*$$
$$= aq - \frac{1}{2}q^2 \Big|_{q=0}^{q=q^*} - p^*q^*$$

which simplifies to

$$CS(p^*, q^*) = \frac{1}{2b}(a - p^*)^2 = \frac{b(q^*)^2}{2}$$

  - Consumer surplus is the area under the inverse demand curve and above the market price



- Prior to looking at the monopolists problem we first consider the competitive case as a benchmark
  - Competitive behavior means that each firm acts as a price taker and chooses quantity
  - The firm assumes that given the price, any quantity that it provides,
    - will be bought
    - will have no effect on the market price
  - Realistic in cases where a firm is very small but not in markets with few large firms
  - So how will a competitive firm choose quantity given price (and how is the price itself determined)?

- Suppose each firm has an increasing cost function $C(q)$ with increasing or constant marginal costs $C'(q)$ (note $C'(q)$ means $dC(q)/dq$)
- The firm maximizes its profits by choosing a *quantity given the fixed price* p. So its maximization problem is given by:

$$\max_q \Pi(q) = qp - C(q)$$

  - To solve this problem, we take the derivative with respect to q, set it to zero and solve for q in that equation (called first order condition or FOC henceforth)
  - Thus FOC gives $p - C'(q) = 0$, and hence a price taking competitive firm will produce at $q = q^e$ where the marginal cost equals the given price p

$$p = C'(q^e) = mc(q^e)$$

  - If the market price increases, a profit maximizing firm will increase the quantity provided

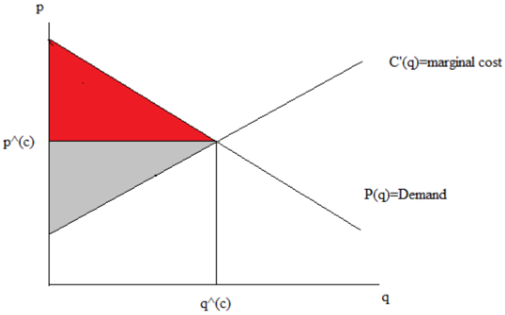
- So how is the price determined in the competitive case?
- This question is dealt with by the concept of competitive equilibrium, which requires that the price be such that the market clears, i.e. *supply equals demand*
- Given an aggregate demand function $Q(p)$ and $n \geq 1$ firms each choosing a quantity q_i a **competitive equilibrium** is a vector

$$(p^e, q_1^e, .., q_n^e)$$

such that

  - (1) q_i^e maximizes $\Pi(q_i) = q_i p^e - C(q_i)$ given p^e for each firm i (price taking behavior)
  - (2) $Q(p^e) = q_1^e + ... + q_n^e$ (market clearing, i.e. aggregate demand equals total supply given p^e)

- An illustration for the case of linear demand $Q(p) = \frac{a}{b} - \frac{p}{b}$ and a unique firm
- Here the competitive price p^e is such that
  - 1) price=marginal cost given $Q(p^e)$
  - 2) the market clears (supply=demand)



(note: in the figure superscript ‘c’ should be ‘e’)

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- How is price/quantity determined under monopoly?
- Say the firm's cost function is $C(q)$ and marginal cost $C'(q)$ is either constant or increasing
  - The monopolist's objective is to maximize profit and faces a downward sloping inverse demand curve $P(q)$ – then the maximization problem is

$$\max_q \Pi(q) = qP(q) - C(q)$$

  - Profit function $\Pi(q)$ is concave in q and the FOC imply

$$P(q) + qP'(q) - C'(q) = 0,$$

    which rewrites as

$$P(q) + qP'(q) = C'(q)$$

- Solving the monopolist's problem gives

$$P(q) + qP'(q) = C'(q)$$

  - The LHS is the marginal revenue from infinitesimally increasing quantity at q
  - The RHS $C'(q)$ is the marginal cost from infinitesimally increasing quantity at q
  - Thus the monopolist produces where

$$mr(q^*) = mc(q^*)$$

  - Reorganising the equality and dividing both sides by $P(q)$ we obtain the *monopoly pricing formula*

$$\underbrace{\frac{P(q) - C'(q)}{P(q)}}_{\text{Markup or Lerner Index}} = \underbrace{\frac{-qP'(q)}{P(q)}}_{\text{inverse price elasticity of demand}} = -\frac{1}{\eta(q)}$$

  - On the LHS is the markup, which is the price-cost difference as a percentage of the price, on the RHS is elasticity of demand in the denominator

- The monopoly pricing formula is

$$\underbrace{\frac{P(q)-C'(q)}{P(q)}}_{\text{Markup or Lerner Index}} = \underbrace{\frac{-qP'(q)}{P(q)}}_{\text{minus one over price elasticity of demand}} = -\frac{1}{\eta(q)}$$

- So the formula gives a simple relation: *the less price elastic is demand, the greater is the markup*
  - Very intuitive – more dependent/less flexible consumers can be exploited more
  - As demand turns more inelastic (η tends to 0), the markup tends to infinity
  - As demand turns more elastic (η tends to infinity), the markup goes to zero – i.e., in the limit it converges to the same markup as in the case of perfect competition

- Finally, note that for the monopoly case we get the same solution if the monopolist had instead solved for optimal price p

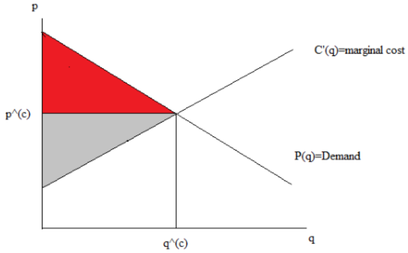
$$\max_p \Pi(p) = Q(p)p - C(Q(p))$$

- In the case of linear demand $p(q) = a - bq$ and if the marginal cost is c then

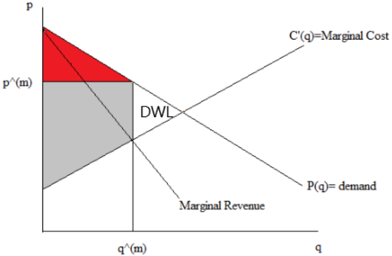
$$p^M = \frac{a+c}{2} \qquad q^M = \frac{a-c}{2b}$$

- Comparison of total welfare
  - Total welfare means considering the welfare of all parties – consumers and firm
  - The loss caused to consumers is larger than the advantage to the firm
  - Monopolistic pricing creates a deadweight loss (DWL)
  - The figures compare competitive outcome (left) to monopolistic one (right) – consumer surplus in red, profit in grey, DWL triangle

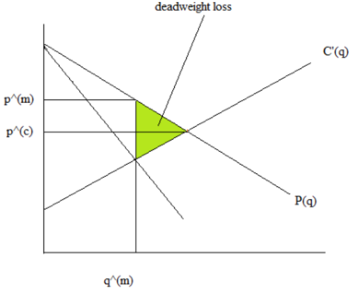
- Competitive Outcome (price = marginal cost)



- Monopolistic Outcome (marginal revenue = marginal cost)



- The welfare cost of monopoly – triangle in green is the reduction in social welfare when moving from competitive pricing to monopoly pricing, i.e., the deadweight loss
  - The DWL loss reflects the *allocative inefficiency* of monopoly
  - How could one try to restore allocative efficiency?
  - Government could try to regulate the monopolist, for example by charging consumers a tax t per unit bought
  - Monopoly vs perfect competition



- A general result: Suppose that the government can regulate the monopolist by setting a per-unit tax paid by consumers – then under general conditions, this tax should be negative, i.e. the government should subsidise consumption of the good which would increase the output from monopoly to competitive level

- There is reason to think that the allocative cost of monopoly is not the only cost associated with it
- Rent Seeking – monopolies also spend a lot of resources to keep or increase their monopoly power (lobbying efforts)
  - These rent seeking activities divert resources from productive activity
  - A firm that has a rent would be willing to dedicate resources to protect it – it might end up dissipating its entire rent (monopoly profit) in the process!
  - See Posner (JPE, 1975), "The social costs of monopoly and regulation" — the term "rent seeking" is due to Krueger (1974, AER) "The political economy of the rent-seeking society"
- Cost distortions/inefficiency – a monopoly may be less prone to keeping costs down
  - No competitive pressure from others to keep costs low
  - Another mechanism – the presence of other firms might be necessary for shareholders to evaluate the performance (yardstick competition)
  - This cannot be done in the same way when there are no other players on the market

Durable Goods Monopoly

- Durable goods do not wear out immediately and a consumer can derive utility from them over several periods – e.g. cars, clothes, fridges, books etc. (unlike products that you consume only once, e.g. food, medicine, etc.)
- Not only do consumers choose if or how much to buy, but when to buy depending on their expectations about future prices
  - So consumers might want to carefully optimize the timing of purchase
  - And the monopolist might want to vary the price over time, which is called "intertemporal price discrimination"
- A surprising result
  - The producer of a durable good who can vary the price over time loses some (or all) monopoly power as compared to a case where they cannot vary the price over time
  - Main intuition: the ability to sell at different points in time at different prices means that the monopolist ends up competing against their own future self

- Consider the following simple two periods setup
  - The good produced and used in period 1 may be used again in period 2
  - After period 2 the good becomes obsolete
  - The cost of producing the good is 0
  - Monopolist and consumers have a discount factor of δ and maximize expected discounted utility at each t
  - There is a continuum of consumers with unit demands and per period willingness to pay θ uniformly distributed on [0,1]
  - A consumer has the same valuation θ across both periods
- Side point on demand ...
  - (For a single period) If there is a continuum of consumers with unit demand and willingness to pay θ uniformly distributed on [0,1], and a monopolist sets a price p, then only those consumers will buy for whom $\theta > p$
  - Thus the demand is given by

$$D(p) = \int_{\theta=p}^{1} f(\theta)d\theta = \int_{\theta=p}^{1} 1d\theta = \theta\Big|_{\theta=p}^{1} = 1 - p$$

- Suppose that consumers are naive and think that the monopolist will charge the same price in both periods

- A consumer with valuation θ will be willing to buy in period 1 if $p \leq \theta(1+\delta)$ i.e., $\theta \geq \frac{p}{1+\delta}$ (because if the consumer buys in period 1 they can enjoy the product in both periods)
  - Then in period 1 monopolist faces demand $D(p) = 1 - \frac{p}{(1+\delta)}$
  - If the monopolist faces naive consumers, they can set price such that period one profit is maximized

    $$p\left(1 - \frac{p}{1+\delta}\right)$$

    which yields the optimal price $p^* = \frac{1+\delta}{2}$
  - With this price p^* in the first period, all consumers with $\theta \geq \frac{1}{2}$ buy

- In period 2 the monopolist now faces a residual demand given by $D(p_2) = \frac{1}{2} - p_2$ as consumers with $\theta < \frac{1}{2}$ have not yet bought anything
  - Facing such a residual demand, the monopolist's best pricing policy in period 2 would be to set p_2 so as to maximize

$$p_2(\frac{1}{2} - p_2)$$

    which yields

$$p_2^* = \frac{1}{4}$$

- But note that in retrospect, some of the consumers who naively bought in period 1 should have waited and bought in period 2 as the price is now lower than in period 1

- Consider for example a consumer with valuation θ given by $\frac{1}{2}+\epsilon$ (note that this is a consumer who bought in period one as those with $\theta \geq \frac{1}{2}$ bought)
  - By buying in period 1 at $p^* = \frac{1+\delta}{2}$, the consumer obtained surplus (over two periods)

$$\epsilon\left(1+\delta\right)$$

  - If they had waited and bought in period 2, the consumer surplus would have been

$$\delta(\frac{1}{4}+\epsilon)$$

  which the latter is larger for small enough ϵ

- Summarizing
  - Non-naive (i.e. sophisticated) consumers will anticipate that the price will decrease in period 2
  - So if the monopolist wants to sell to sophisticated consumers in period 1, they will have to lower their price in period 1

- So what is an equilibrium going to look like under intertemporal price discrimination?
  - The price goes down over time
  - Only high valuation consumers buy in the first period at the high price – given their high valuation, they are better off getting the good early
  - Consumers with lower valuations prefer to wait
- The equilibrium rests on a central *incentive constraint* – the first period price cannot be too high, otherwise some high valuation consumers start deferring acquisition
- Does the ‘decreasing price’ prediction resonate with anything we observe in practice?
  - Early hardcover prints of a book are typically more expensive compared to paperbacks released few months later
  - Cost of hardback vs paperback is not so different (distribution is the same) so costs are not the reason – arguably intertemporal price discrimination

- Using the same set up as before and solving the problem where the monopolist maximizes the joint profit over two periods we can show the following result (derivation/proof omitted)
- *In the two period durable goods problem with a continuum of consumers and without price commitment, the monopolist obtains lower profit and sets a lower first-period price than in a situation in which they would exogenously be restricted to charging the same price in both periods*
- So when then monopolist has the ability to change their price over time
  - Period 1 price is lower than if the monopolist could commit
  - Price goes down over time
  - The monopolist makes less profit
  - In seminal papers on the topic, authors find that under general conditions, the profits of the monopolist tend to 0 under no price precommitment – so the monopolist entirely loses their monopoly power!

- How could the monopolist avoid this problem?
  - Renting or leasing instead of selling
    - the good is no longer bought but effectively returned to the monopolist at the end of each period – then the monopolist can realize the monopoly price in each period
    - there are moral hazard problems (downside) associated with leasing – customers may misbehave and damage the good that is being leased thereby causing losses to the monopolist
  - Committing to a sequence of prices
    - reputation for never decreasing price – this is apparently the case of the diamond producer DeBeers
    - money-back guarantees – the monopolist might commit to paying back period 1 customers if period 2 prices decrease – by doing this, the monopolist eliminates incentive to decrease price in future
  - Planned obsolescence
    - decrease the durability of the durable good
    - this reduces the quantity of the good that is carried over from one period to the next
    - by making tomorrows good scarce, it can be shown that the monopolist effectively commits to setting a high price tomorrow – this in turn allows them to set a high price today

Introduction to Games

- Between monopoly pricing and a scenario where multitudes of firms interact, there is an intermediate case – a limited number of firms competing with each other where each firm's actions has an impact on the market outcome
- This is the case of oligopolistic markets – here firms take into account the actions of their competitors before choosing their own actions – for instance what price and/or quantity to set
- Firms' decisions and market outcome are often modeled using tools from game theory – in this part, we introduce some basic concepts from game theory and then turn to oligopoly theory after that

- In a game, different rational agents interact and make decisions in order to maximize their payoff – each of their decisions also affects the payoffs (and hence decisions) of other agents $\rightarrow$ **decisions are interdependent** and game theory is the formal study of such situations
- Game theory models situations when an agent's utility/profit is affected by others' choices
- Types of games

- Types of games
  - Simultaneous move games (normal-form games):
    - modelling: players, normal-form, strategy, payoffs, etc.
    - solution concepts: dominant strategy equilibria, Nash equilibria, mixed strategy Nash equilibria
  - Sequential move games
    - modelling: game tree, extensive-form, subgames, strategy
    - solution concepts: As above + subgame perfect equilibria, backward induction
  - Imperfect information games:
    - modelling: information sets, moves by nature
    - solution concepts: as above

- A **player** is an agent who interacts – for example, firms like Coke and Pepsi $I = \{1, 2, \ldots, N\}$ is a set of $N > 1$ players
- A **strategy** is a choice for a player – e.g. advertise or not, go left or right
A strategy set S_i for player i is a list of all possible strategies $S_i = \{\text{left}, \text{right}\}$
- A **strategy profile** consists of strategies of all players $s = (s_1, s_2, \ldots, s_N)$ – it is a set of strategies for all players which fully specifies all actions in a game (and it must include one and only one strategy for every player)
  - $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots s_N)$ is the strategy profile of all players except that of i, so strategy profile is also written as $s = \{s_{-i}, s_i\}$
  - the cartesian product of all S_i denoted $\Pi_{i \in I} S_i$ is the set of strategy profiles – e.g. with two players with strategies 'left' and 'right' it is {(left, left), (left, right), (right, left), (right, right)}
- A **payoff** is a reward (utility/profit) of a particular strategy profile for example, the profit each firm gains after each of them make their own independent decision of whether to advertise

- Player **payoffs** are defined over strategy profiles – a strategy profile implies an outcome of a game
  - Player i's payoff from the strategy profile s is $u_i(s)$
  - Alternatively, player i's payoff if she chooses s_i and others play s_{-i} is given by $u_i(s_i, s_{-i})$
- A **best response** is a strategy s_i (among all possible strategies) of a player that produces the highest payoff u_i for her given the strategy employed by the other player(s)
  - Formally, a strategy s_i is a **best response** by player i to a profile of strategies for all other players s_{-i} if

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

for all $s_i' \in S_i$

- A central solution concept we use in games is the Nash equilibrium – it is a strategy profile where no individual has a unilateral incentive to change their behavior

- Consider a game of N players. A **Nash equilibrium** is a strategy profile $s^* = (s_1^*, s_2^*, \dots, s_N^*)$ if for every player i

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*)$$

  for all $s_i' \in S_i$

- Equivalently, $s^* = (s_1^*, s_2^*, \dots, s_N^*)$ is a Nash equilibrium if s_i^* is a best response to s_{-i}^* for all $i = 1, 2, \dots, N$

- Thus if each player employs their strategy in a way such that no players have an incentive to deviate from it, then the existing strategies constitute a Nash equilibrium:
  - You choose your strategy, and I choose mine
  - Then we look at our respective payoffs
  - I see that given your strategy, I cannot improve my payoff even if I deviate from what I have already chosen
  - You see that given my strategy, you cannot improve your payoff even if you deviate from what you have already chosen
  - We do not change our strategies
  - This is a Nash equilibrium
- We now look at some examples to make these concepts more concrete starting with simultaneous (normal-form) games

- Two suspects held in separate cells are charged with a major crime. However, there is not enough evidence.
- Both suspects are told the following policy
  - If neither confesses, then both will be convicted of a minor offense and sentenced to one year in jail
  - If both confess, then both will be sentenced to jail for five years
  - If one confesses, but the other does not, then the confessor will be released but the other will be sentenced to jail for twenty years!
- How should they play?

- The payoff matrix is as follows
  - Row and column players payoffs are x and y respectively and written as (x, y)

		Player B	
		Confess	Silent
Player A	Confess	(-5,-5)	(0,-20)
	Silent	(-20,0)	(-1,-1)

  - What is the best response of A when B confesses?
    - if A confesses then A will be in jail for 5 years
    - if A keeps silent A will be in jail for 20 years
    - so, A's best response – maximizing her payoff – is to confess
  - What is the best response of A when B remains silent?
    - if A confesses then A will be free!
    - if A keeps silent they A will be in jail for 1 year
    - A's best response, again, is to confess
  - Similarly, regardless of A's strategy, best response of B is to confess
  - Unique Nash Equilibrium (NE) of this game is (confess,confess)

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    - if A confesses then A will be free!
    - if A keeps silent they A will be in jail for 1 year
    - A's best response, again, is to confess
  - Similarly, regardless of A's strategy, best response of B is to confess
  - Unique Nash Equilibrium (NE) of this game is (confess,confess)

- The payoff matrix is as follows
  - Row and column players payoffs are x and y respectively and written as (x, y)

		Player B	
		Confess	Silent
Player A	Confess	(-5,-5)	(0,-20)
	Silent	(-20,0)	(-1,-1)

  - What is the best response of A when B confesses?
    - if A confesses then A will be in jail for 5 years
    - if A keeps silent A will be in jail for 20 years
    - so, A's best response – maximizing her payoff – is to confess
  - What is the best response of A when B remains silent?
    - if A confesses then A will be free!
    - if A keeps silent they A will be in jail for 1 year
    - A's best response, again, is to confess
  - Similarly, regardless of A's strategy, best response of B is to confess
  - Unique Nash Equilibrium (NE) of this game is (confess,confess)

- Note that ‘confess’ is the **dominant strategy** in the prisoner’s dilemma
- This means irrespective of the other player’s strategy, the payoff from playing Confess dominates the payoff from playing Silent
- Hence (confess, confess) is a dominant strategy equilibrium
- The prisoner’s dilemma applies to many other situations
  - Branding and Advertising
  - Patent races, etc
- Equilibrium in other games might not be a dominant strategy equilibrium
- We now provide examples of such games

- Two teenagers driving on a narrow road in opposite directions
  - Neither of them wants to move out of the way
  - Whoever ‘chickens’ out loses their pride (gets 0), and the tough one wins (gets 10)
  - If both play tough then they break their bones (both get -10)!
  - If both play chicken then only their pride is slightly damaged (both get -5)

		Player 2	
		Tough	Chicken
Player 1	Tough	(-10,-10)	(10,0)
	Chicken	(0,10)	(-5,-5)

- Two pure strategy Nash equilibria (Tough, Chicken) and (Chicken, Tough)

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- Two pure strategy Nash equilibria (Tough, Chicken) and (Chicken, Tough)

- A prisoner can either climb a wall to escape or dig a tunnel in cell to escape
  - A warden can either guard the wall or inspect the cell
  - If they both are in the same place then the prisoner gets caught and punished and the warden gets the reward – the opposite happens if they are not in the same place
  - Payoffs are as given below

		Warden	
		Guard wall	Inspect cell
Prisoner	Climb wall	(-1,1)	(1,-1)
	Dig tunnel	(1,-1)	(-1,1)

- Cell by cell inspection shows no equilibrium (in pure strategies)

- A **pure strategy** is a strategy that a player plays with a 100% probability
  - for instance, when a player plays 'Confess' in prisoner's dilemma, they play that strategy for sure – and hence Confess, for them, is a pure strategy
- A **mixed strategy** is an assignment of probability to each pure strategy – this allows a player to randomly select a pure strategy
- It can be shown in the Prisoner-Warden game that although there is no pure Nash strategy equilibrium, there is a mixed strategy equilibrium in which
  - the prisoner chooses climbing a wall with 50% probability and digging tunnel with a 50% probability
  - whereas the warden chooses to guard the wall with a 50% probability and watching cells with a 50% probability
  - Note that 50-50 probability is because of the values of payoffs chosen (and number of strategies) and will not be always 50-50

- Suppose the prisoner (p) plays strategy ‘climbing wall’ with probability x, and of ‘dig tunnel’ with probability $(1-x)$
- Similarly, say the warden (w) plays the strategy ‘guard wall’ with probability y, and ‘inspect cells’ with probability $(1-y)$
- How do we compute these probabilities?
  - Start with p – then the expected payoff for p if she decides to climb will be $y(-1)+(1-y)(1)$ and the expected payoff if she digs will be $y(1)+(1-y)(-1)$
  - So p will climb if $y(-1)+(1-y)(1) > y(1)+(1-y)(-1)$ and will dig if $y(-1)+(1-y)(1) < y(1)+(1-y)(-1)$
  - However, since w wants to maximize his payoff, he should take action to minimize the payoff to p – he can do that by picking y so that $y(-1)+(1-y)(1) = y(1)+(1-y)(-1)$, which solves for $y = 1/2$

  - By symmetric argument, $x = 1/2$

- Lets consider a three strategy game
  - Player 1 can move U,M and D
  - Player 2 can move L,C,R
  - The payoffs are as given in the table

		Player 2		
		L	C	R
Player 1	U	(3,2)	(0,3)	(2,0)
	M	(1,3)	(2,0)	(1,2)
	D	(2,1)	(4,3)	(0,2)

  - Equilibrium (D,C)

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- Equilibrium (D,C)

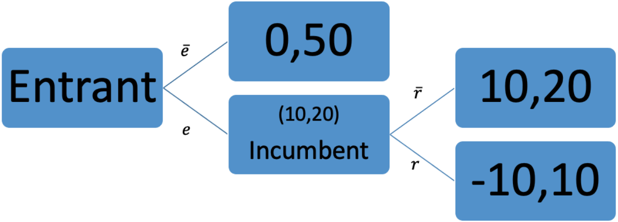
- So far we have seen examples of simultaneous move games with finite discrete strategies
  - Many strategies are continuous in real life: how much to produce, what price to charge, what level of investment to make, how much to bribe, how much to bid $\cdots$
  - We will later consider the case of firms choosing price and quantity
- We next turn to **sequential (extensive-form) games**
  - Also known as tree-form games
  - Decision nodes indicate the player whose turn it is to move (rules)
  - Branches denote possible choices
  - End node indicates each player's payoff (by order of appearance)
  - Games are solved by backward induction

- Consider the following setup
  - There is an incumbent and a potential entrant in the market
  - If the potential entrant decides to enter, the incumbent threatens to retaliate (for instance by cutting prices significantly)
  - The payoffs under different combinations of enter/not enter and retaliate/not retaliate are given below

		Incumbent	
		Retaliate	Not retaliate
Entrant	Enter	-10,10	10,20
	Not enter	0,50	0,50

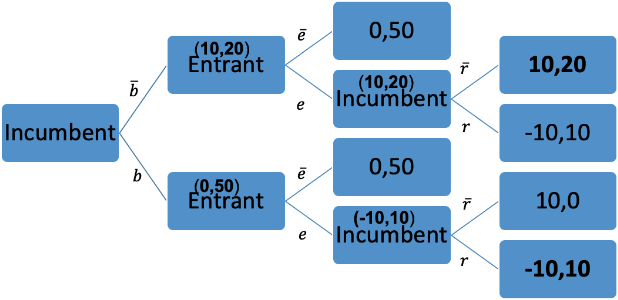
- There are two Nash equilibria of the game — are both reasonable?

- First the entrant chooses to enter or not, then the incumbent decides if she should execute the threat or not



- If it is known that the incumbent is rational, then r is not a **credible** threat
- So the potential entrant will enter and the incumbent will not retaliate
- More generally, define a subgame as the game corresponding to a subtree
- A **subgame perfect equilibrium** (SPE) is one such that the equilibrium strategies form an equilibrium at every subgame

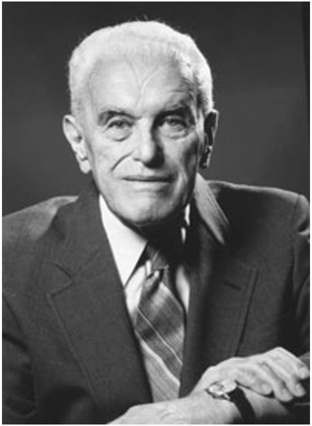
- Say the incumbent can **commit** to retaliation (e.g. by investing in capacity) – then the outcome can change



- Value of commitment
  - Incumbent's equilibrium payoff in first entry game: 20
  - Incumbent's equilibrium payoff in new entry game: 50
  - Value of commitment: 50-20=30

- ‘Game theory is good for solving mathematical problems, but not for playing games · · · ” Reinhard Selten
  - Reinhard Selten (1994 Nobel Memorial Prize in Economics, for his contributions in the area of game theory (with John Nash and John Harsanyi)

- So far, we have been assuming that everything in the game was common knowledge for everybody playing
  - But in fact, players may have private information about their own payoffs, about their type or preferences, etc.
  - The way to modelling this situation of asymmetric or incomplete information is by referring to an idea generated by Harsanyi (1967)
  - The key is to introduce a move by **Nature**, which transforms the uncertainty by converting an *incomplete information* problem into an *imperfect information* problem



- Example of a Bayesian game
  - Nature decides whether the payoffs are as in Matrix I or Matrix II, with equal probabilities
  - ROW player is informed of the choice of Nature, and COL is not
  - ROW chooses U or D, and COL chooses L or R (choices are made simultaneously)
  - Find all the Bayesian Nash equilibria

	Matrix I	
	L	R
U	1,1	0,0
D	0,0	0,0

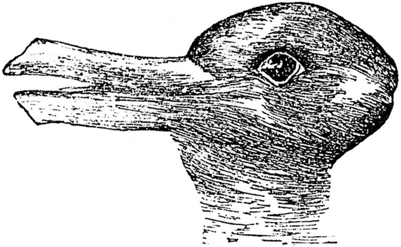
	Matrix II	
	L	R
U	0,0	0,0
D	0,0	2,2

- Find all the Bayesian Nash equilibria
  - First, determine the strategies for each player
  - COL has only two strategies (L and R) because they do not know in which matrix the game is played
  - ROW knows in which Matrix the game occurs, and the strategies are UU (play U in case of being in Matrix I and U in case he is in Matrix II), UD, DU, and DD
  - Knowing the probability (a half) Nature locate the game in each matrix, the new extended game is given below

	L	R
UU	1/2, 1/2	0,0
UD	1/2, 1/2	1,1
DU	0,0	0,0
DD	0,0	1,1

  - DU is a dominated strategy for ROW. After eliminating that possibility, the game has three pure-strategy Nash Equilibria: (UU,L), (UD,R), and (DD,R)

- Often, however, game theory results do not predict the outcome in real life—say in the market ···
- Because the assumption of ‘rationality’ is not always correct
- Players view of the same object differs and they play differently
- Evolutionary game theory studies games where agents do not need to be rational



- Between monopoly pricing and a scenario where multitudes of firms interact, there is what might be called an intermediate case – a limited number of firms competing with each other
- This is the case of oligopolistic markets – here firms take into account the actions of their competitors before choosing their own actions – for instance what price and/or quantity to set
- There are different models of oligopoly – we will focus on three specific models
    - **Cournot**: firms decide quantity, and price adjusts to consumer demand (automobiles?)
    - **Stackelberg**: firms decide quantity sequentially, and price adjusts to consumer demand. There is a leader and a follower.
    - **Bertrand**: firms set prices and sell whatever is demanded at those prices (most services)
    - Our initial focus will be on homogenous goods duopoly markets – but will then generalize the results to case of more than two firms (still homogenous in this lecture)

- There are 2 firms, $i = 1, 2$ each producing the same homogeneous good
- Firms simultaneously set their quantities q_i and the cost functions are given by

$$TC_i(q_i) = c_i(q_i) \qquad c_i \geq 0$$

- Inverse demand function is $P(Q)$, where Q is the total industry output and $p(Q)$ is linear in Q and given by

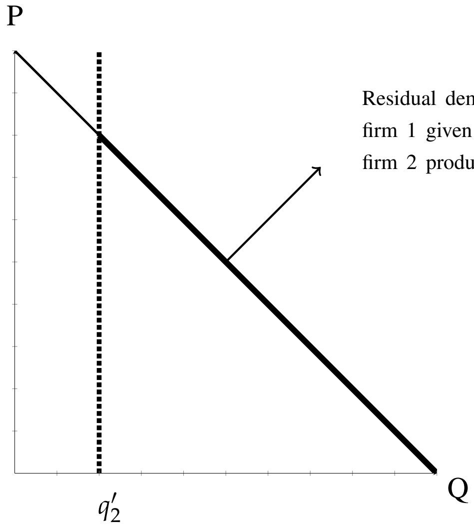
$$p(Q) = a - bQ, \qquad a, b > 0 \qquad Q = q_1 + q_2$$

- $P(q)$ is thus the price at which the full quantity q is demanded (absorbed by) consumers
- We can analyze this as a 2-player simultaneous move game in quantity and look for the Nash equilibrium

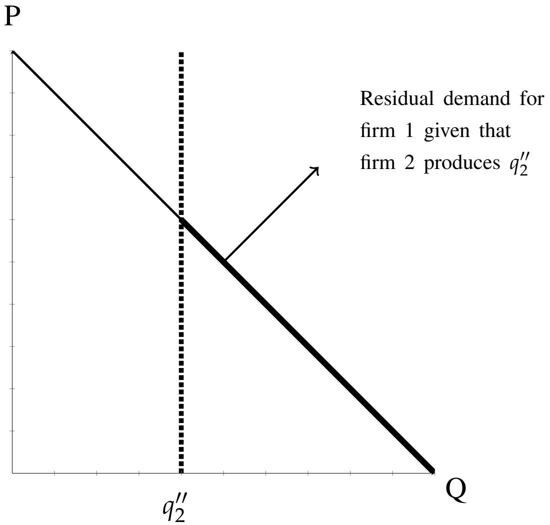
- Description of the game
  - Firms choose quantity simultaneously – each firm chooses $q_i \in [0, \infty)$
  - The payoff function for each firm (player) is its profit

$$\Pi_i(q_1, q_2) = p(q_1 + q_2)q_i - c_i(q_i)$$

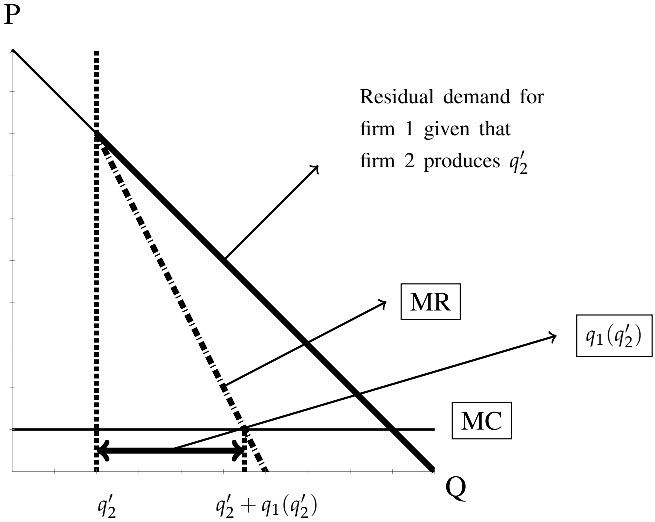
- We now need to define an equilibrium concept
- The triplet $\{p^c, q_1^c, q_2^c\}$ is a **Cournot-Nash Equilibrium** if
  - Given firm 2 chooses $q_2 = q_2^c$, firm 1's best response is q_1^c (thus q_1^c solves $\max_{q_1} \Pi_1(q_1, q_2^c)$)
  - Given firm 1 chooses $q_1 = q_1^c$, firm 2's best response is q_2^c (thus q_2^c solves $\max_{q_2} \Pi_2(q_1^c, q_2)$)
  - $p^c = a - b(q_1^c + q_2^c)$ and $p^c, q_i^c \geq 0$
- Lets first look at this problem graphically



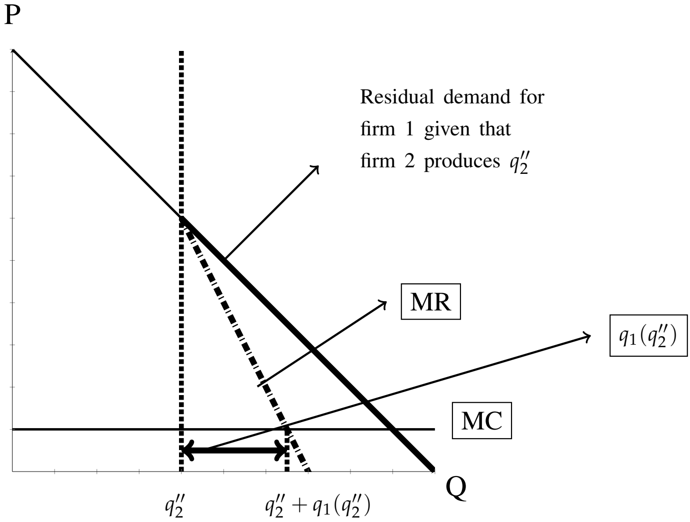
P
Residual demand for firm 1 given that firm 2 produces q_2'
q_2'
Q



P
Residual demand for firm 1 given that firm 2 produces q_2''
Q
q_2''

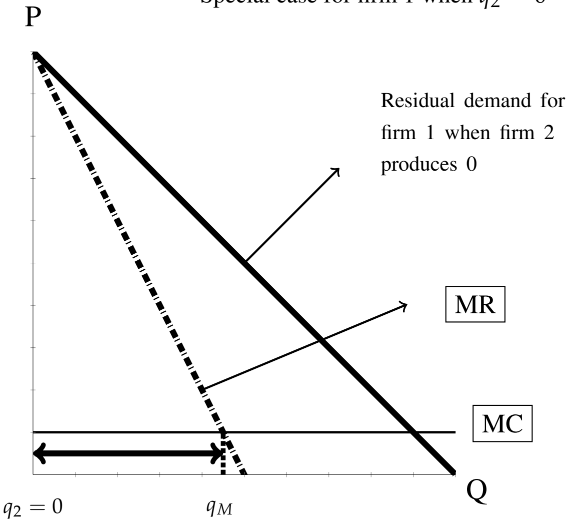


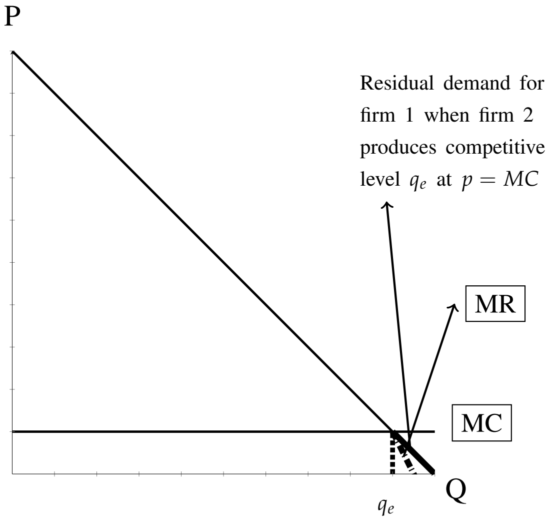
P
Residual demand for
firm 1 given that
firm 2 produces q_2'
MR
$q_1(q_2')$
MC
Q
q_2'
$q_2' + q_1(q_2')$



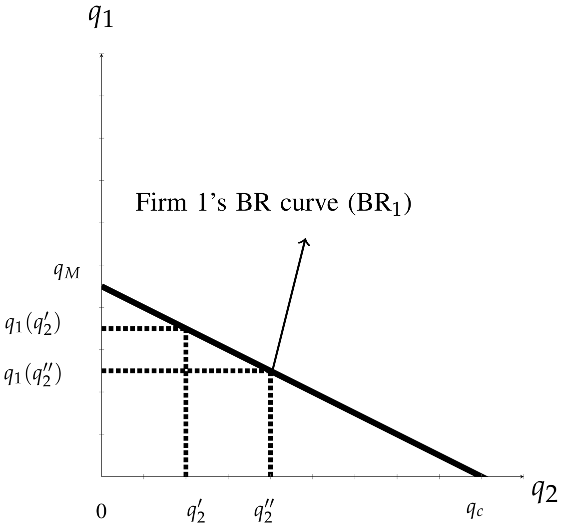
P
Residual demand for firm 1 given that firm 2 produces q_2''
MR
$q_1(q_2'')$
MC
Q
q_2''
$q_2'' + q_1(q_2'')$

Special case for firm 1 when $q_2 = 0$

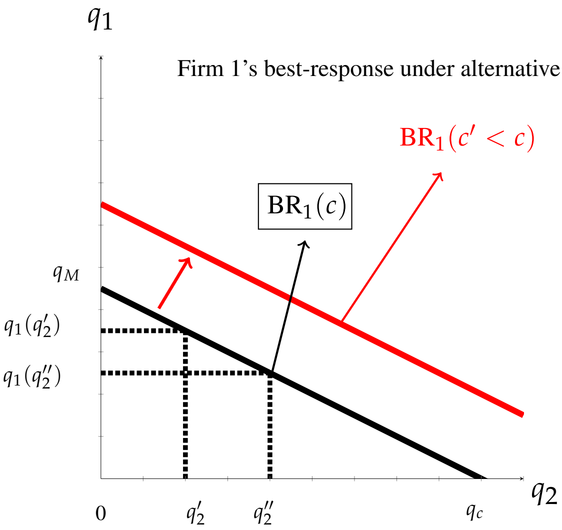




P
Residual demand for firm 1 when firm 2 produces competitive level q_e at $p = MC$
MR
MC
Q
q_e



q_1
Firm 1's BR curve (BR_1)
q_M
$q_1(q_2')$
$q_1(q_2'')$
0
q_2'
q_2''
q_c
q_2



q_1
Firm 1's best-response under alternative marginal costs
$\mathrm{BR}_1(c' < c)$
$\mathrm{BR}_1(c)$
q_M
$q_1(q_2')$
$q_1(q_2'')$
0
q_2'
q_2''
q_c
q_2

- Return now to the two-firm game set up initially
  - Two firms, $i = 1, 2$, with cost functions $TC_i(q_i) = c_i q_i$ with $c_i \geq 0$
  - Linear demand $p(Q) = a - bQ$ and $a, b > 0$, $Q = q_1 + q_2$
  - Payoff functions (profits) given by $\Pi_i(q_1, q_2) = p(q_1 + q_2)q_i - c_i(q_i)$
- We can solve for best response functions and equilibrium
  - Profit for firm 1

$$\begin{aligned}\Pi_1(q_1, q_2) &= p(q_1 + q_2)q_1 - c_1 q_1 \\ &= aq_1 - b(q_1 + q_2)q_1 - c_1 q_1\end{aligned}$$

  - Given q_2, the FOC for profit maximization $\frac{\partial \Pi_1}{\partial q_1} = 0$ gives

$$a - 2bq_1 - bq_2 - c_1 = 0$$

    which can be solved for q_1 as a function of q_2
  - Thus, firm 1's best response function is

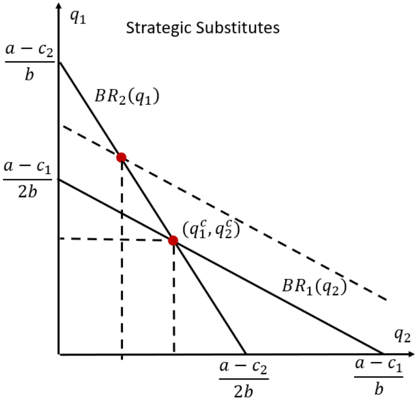
$$q_1 = R_1(q_2) = \frac{a - c_1}{2b} - \frac{q_2}{2}$$

- The best response function of firm 2 can be derived the same way
  - Thus the two best response functions are

$$q_1 = R_1(q_2) = \frac{a - c_1}{2b} - \frac{q_2}{2}$$
$$q_2 = R_2(q_1) = \frac{a - c_2}{2b} - \frac{q_1}{2}$$

  - Compare these best response quantities to the monopoly case – recall that under monopoly with a linear demand curve $p(q) = a - bq$ and marginal cost c, the monopolist sets output to $q_M = \frac{a-c}{2b}$ – thus we can think of $R_i(q_j) = q_M - \frac{q_2}{2}$
  - Similarly, if the competitor sets output to zero, then the remaining firm produces at monopoly level, e.g. if $q_2 = 0$ then $q_1 = \frac{a-c}{2b} = q_M$

- The figure shows the best response functions of the firms
  - **Strategic substitutes**
  - They are downward sloping – if firm 1 increases output, firm 2 decreases its output, if firm 1 decreases output, firm 1 increases output
  - If marginal cost decreases, best response shifts out – in turn that changes the equilibrium
  - Recall: (q_1^c, q_2^c) is a Nash equilibrium if $\Pi_1(q_1^c, q_2^c) \geq \Pi_1(q_1, q_2^c), \ \forall q_1 \neq q_1^c$ & $\Pi_2(q_1^c, q_2^c) \geq \Pi_2(q_1^c, q_2), \ \forall q_2 \neq q_2^c$
  - Solution (Cournot Nash Equilibirum) is where the best response functions intersect at (q_1^c, q_2^c)



- We can solve the two reaction functions for equilibrium
  - These are

$$q_1^c = \frac{a - 2c_1 + c_2}{3b} \qquad q_2^c = \frac{a - 2c_2 + c_1}{3b}$$

  - The aggregate demand and price is

$$Q^c = q_1^c + q_2^c = \frac{2a - c_1 - c_2}{3b}$$
$$p^c = a - bQ^c = \frac{a + c_1 + c_2}{3}$$

  - Firm profit is

$$\Pi_i^c = \frac{(a - 2c_i + c_j)^2}{9b} = b(q_i^c)^2$$

- Price cost margin
  - For firm i, FOC was
$$\frac{\partial \Pi_i}{\partial q_i} = p - c_i + q_i \frac{\partial p}{\partial Q}, \qquad \text{which implies}$$
$$p - c_i = -q_i \frac{\partial p}{\partial Q} = \frac{q_i}{Q}\Big(- \frac{\partial p}{\partial Q}\frac{Q}{p}\Big)$$
$$\frac{p - c_i}{p_i} = \frac{s_i}{\eta}$$

    where $s_i = \frac{q_i}{Q}$ is the share of firm 1 and η is he elasticity of demand
  - So price cost margin for a firm (i.e. its **Lerner index**) is s_i/η
  - How to interpret the above equality?
  - The more relatively efficient an individual firm is (i.e. the higher its market share) and the more inelastic the demand it faces, the more market power it achieves, and the more it can extract profits from the market

- Similarly, we can link these price cost margins to Herfindahl index (HHI), a concentration measure defined as $HHi = \sum_i s_i^2$
  - Recall

$$\frac{p - c_i}{p_i} = \frac{s_i}{\eta}$$

$$\frac{p - c_i}{p_i} s_i = \frac{s_i}{\eta} s_i$$

$$\sum_i \frac{p - c_i}{p_i} s_i = \frac{1}{\eta} \sum_i s_i^2$$

  - The Average Lerner Index (i.e. average markup) is proportional to the Herfindahl index (i.e. our index of market concentration)
- One can interpret the result as showing that the Herfindahl index is a good measure of average market power

- While the model we discussed above is for two firms, it extends to N firms
- In the special case of identical firms ($c_i = c$ and $F_i = F$ for all i) the output for each firm $q_1^c = q_2^c = \ldots = q_N^c$ and the equilibrium outcomes are as follows

$$q^c = \frac{a-c}{(N+1)b} \qquad Q^c = \left(\frac{a-c}{b}\right)\left(\frac{N}{N+1}\right)$$

$$p^c = \frac{a+Nc}{N+1} \qquad \Pi_i^c = \frac{(a-c)^2}{(N+1)^2 b}$$

  - Note that $N \to \infty$, the equilibrium values approach the competitive levels, i.e. $q^c \to 0$ and $p^c \to c = p^e$

- We now turn to a variant of the quantity setting model where firms instead of setting quantities simultaneously move *sequentially*
  - This is referred to as *Stackelberg*, or *Leader-follower* model
  - Here we take the order of moves as given rather than question how one firm becomes a leader
  - Instead the sort of questions we answer are: (i) is there a first mover advantage? (ii) how does the equilibrium compare to the Cournot case discuss earlier?
  - This type of model fits the extensive form game with sequential moves we discussed earlier and the solution concept we will use is the subgame perfect equilibrium

- Model setup
  - Two firms ($i = 1, 2$), firm 1 moves first and sets quantity
  - Firm 2 observes q_1 and sets q_2
  - Firms are otherwise identical ($c_i = c$, and fixed cost is zero for both)
  - Market demand (as before) is $p(Q) = a - bQ$ and $a, b > 0$, $Q = q_1 + q_2$
  - Payoff functions (as before) $\Pi_i(q_1, q_2) = p(q_1 + q_2)q_i - cq_i$
- General method for solution
  - Backward induction starting with period 2 and then period 1
  - In period 2, firm 2 takes q_1 as given and chooses q_2 to maximize its profit
  - In period 1, firm 1 sets q_1 to maximize it's own profit, but it does so anticipating how firm 2 will react in period 2, i.e., it uses firm 2's best response function from period 2 in period 1 to solve its own output

- Period two subgames
  - In the second period, firm 2 chooses output q_2 given q_1
  - This problem is the same problem that firm 2 faced in the simple Cournot case, which we have already solved and know the best response function of firm 2 (so we don't need to solve it again)

$$q_2 = R_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$$

- Period one subgame
  - Firm 1 maximizes its profit via FOC

$$\max_{q_1} \Pi_1 = p(q_1 + q_2)q_1 - cq_1$$

  - However, firm 1 can also calculate firm 2's best response $R_2(q_1)$ and substitutes that in the maximization problem above

$$\begin{aligned} \max_{q_1} \Pi_1 &= p(q_1 + R_2(q_1))q_1 - cq_1 \\ &= \left[a - b\left(q_1 + \frac{a-c}{2b} - \frac{q_1}{2}\right)\right]q_1 - cq_1 \end{aligned}$$

- The FOC give the following solution

$$q_1^s = \frac{a-c}{2b} \qquad q_2^s = \frac{a-c}{4b}$$
$$\Pi_1^s = \frac{(a-c)^2}{8b} \qquad \Pi_2^s = \frac{(a-c)^2}{16b}$$
$$p^s = \frac{a+3c}{4} \qquad Q^s = \frac{3(a-c)}{4b}$$

  - Note that $q_1^s > q_2^s$ and following from that $\Pi_1^s > \Pi_2^s$
  - The advantage (first mover) for firm 1 is due to the slope of best response functions, which were downward sloping (strategic substitutes) – if firm 1 expanded its output, firm 2 responded by reducing it

- How does the Stackelberg model solution compare to the Cournot model?

$$q_1^s = \frac{3}{2}q_1^c > q_1^c \quad q_2^s = \frac{3}{4}q_2^c < q_2^c$$
$$\Pi_1^s > \Pi_1^c \qquad \Pi_2^s < \Pi_2^c$$
$$p^s < p^c \qquad Q^s > Q^c$$

  - Compared to the static Cournot model firm 1's output and profit is greater under the Stackelberg leader-follower model while it is the other way around for firm 2
  - Price is lower and total quantity produced higher under Stackelberg relative to Cournot — consumer welfare higher

- We finally turn to the case when firms compete in prices – referred to as the Bertrand Competition
- In this model, firms set prices rather than output
- Attractive feature of price competition model is that firms are able to change prices faster and at lower cost than output, as changing the latter may require changes in inventory and capacity
- Thus in the short run quantity changes may not be feasible

- As before there are two firms with cost functions given by $TC_i(q_i) = c_i q_i$ where $c_i \geq 0$ (constant marginal costs and zero fixed costs) and industry demand is $p(Q) = a - bQ$ and $a, b > 0$

- Unlike the previous case with a single market price, firms can set different prices

- Thus we make the following explicit assumption about consumer behavior

  - Consumers buy from the cheapest seller
  - If prices are the same, firms spilt the market (i.e., half of the consumers buy from firm 1 and others from firm 2)

- Formally the rationing rule is

$$q_i = \begin{cases} 0 & \text{if } p_i > a \\ 0 & \text{if } p_i > p_j \\ \frac{a-p}{2b} & \text{if } p_i = p_j = p < a \\ \frac{a-p_i}{b} & \text{if } p_i < \min\{a, p_j\} \end{cases}$$

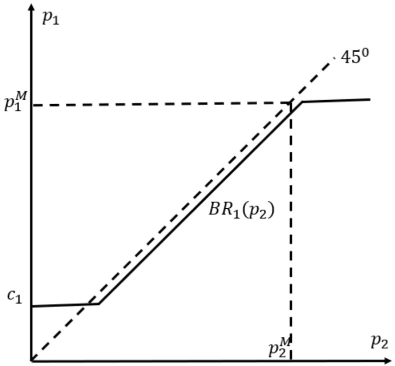
- Firms choose prices simultaneously and noncooperatively
- A pure strategy Nash Equilibrium is a pair of prices (p_1^b, p_2^b) such that each firm maximizes its profits given the other firm's price (i.e., no incentive to unilaterally change price) and where outputs (q_1^b, q_2^b) are per the ration rule given above
- Thus a quadruple $\{p_1^b, p_2^b, q_1^b, q_2^b\}$ is a **Bertrand Nash Equilibrium** if
  - Given firm 2 chooses $p_2 = p_2^b$, firm 1's best response is p_1^b (thus p_1^b solves $\max_{p_1} \Pi_1(p_1, p_2^b)$)
  - Given firm 1 chooses $p_1 = p_1 b$, firm 2's best response is p_2^b (thus p_2^b solves $\max_{p_2} \Pi_2(p_1^b, p_2)$)
  - q_1^b, q_2^b are determined per the rationing rule given earlier
- More succinctly, a pair of prices $\{p_1^b, p_2^b\}$ is a Bertrand Nash Equilibrium if

$$\Pi_i(p_i^b, p_j^b) \geq \Pi_i(p_i, p_j^b), \qquad \text{for all } i = 1,2 \text{ and } p_i \neq p_i^b$$

- To understand how equilibrium will be established, let's look at the best response functions

- To understand how equilibrium will be established, let's look at the best response functions (starting with firm 1)
  - Consider first the case $c_1 = c_2$
  - If $p_2 < c$, then firm 1 sets $p_1 = c$
  - If $p_2 > c$ but below monopoly price $p_1^M = p_2^M = p^M$, then firm 1 undercuts firm 2 by a small amount ϵ
  - When firm 2 sets prices above monopoly prices p^M, firm 1 sets price at the monopoly level
- Next, superimpose firm 2's reaction function

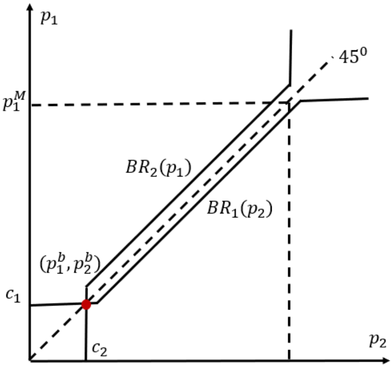
- The figure shows the BR function of firm 1 (upward sloping)



- To understand how equilibrium will be established, let's look at the best response functions

- To understand how equilibrium will be established, let's look at the best response functions (superimpose firm 2's BR)
  - Consider first the case $c_1 = c_2$
  - Because $c_1 = c_2$, BR_2 is symmetrical
  - If $p_1 > c$ but below monopoly price $p_1^M = p_2^M = p^M$, then firm 2 undercuts firm 1 by a small amount ϵ
  - When firm 1 sets prices above monopoly prices p^M, firm 2 sets price at the monopoly level
- The BR's intersect at $p_1 = p_2 = c$ — this is the Bertrand Nash equilibrium

- BR of firm 1 & 2 (upward sloping) – **strategic complementarity**



- Thus we have the following results
  - When marginal costs are identical ($c_1 = c_2 = c$), the unique Nash equilibrium is
    - $p_1^b = p_2^b = c = p^e$
    - Firms set price equal to marginal cost as in the competitive case – Bertrand's paradox: just two firms are enough to give a competitive outcome
    - Firms make zero profit – if there were additional fixed cost $F > 0$, firms would make a loss!
- The model can be extended to the case with $c_1 \neq c_2$
  - Say $c_1 < c_2$ and $c_2 - c_1 > \epsilon$
  - Then $p_2^b = c_2, p_1^b = c_2 - \epsilon$ and $q_1^b = (a - c_2 + \epsilon)/b$ and $q_2^b = 0$
  - In words: the more efficient firm 1 sets its price equal to the marginal cost of the less competitive firm and makes a positive profit, and firm 2 does not produce anything

- For the model with equal marginal cost, we get the Bertrand paradox
  - We call this a paradox because it is hard to believe that duopoly firms (only 2 firms!) would make no profits
  - How robust is this result to modelling assumptions (e.g. homogeneous costs, known costs)?
  - There are a variety of alternatives to this simple setup, for which this radical result breaks down
    - uncertain costs
    - capacity constraints
    - product differentiation (we will consider this in another lecture)

The End